

Helicity Evolution at Small x

Matthew D. Sievert
with Daniel Pitonyak
and Yuri Kovchegov

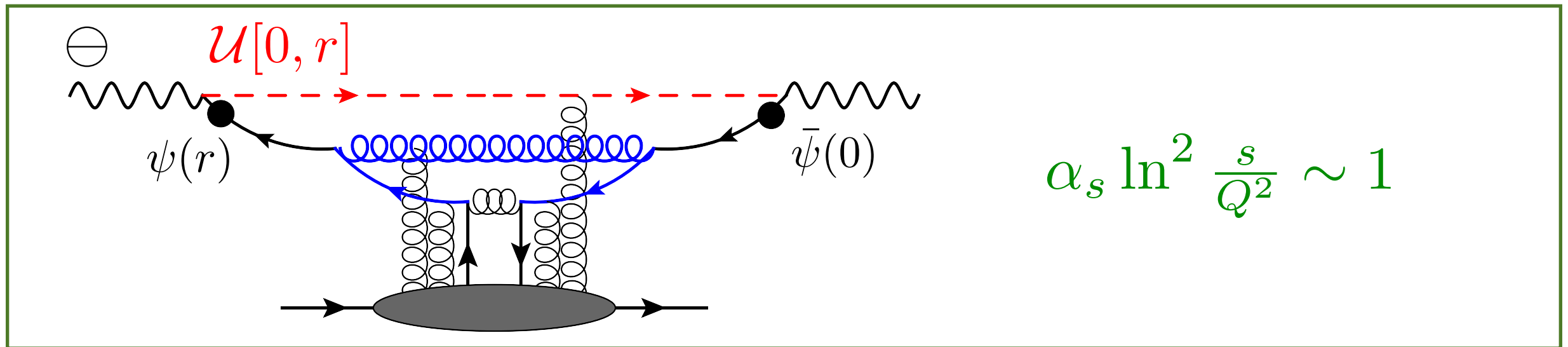


Tuesday Feb. 9, 2016

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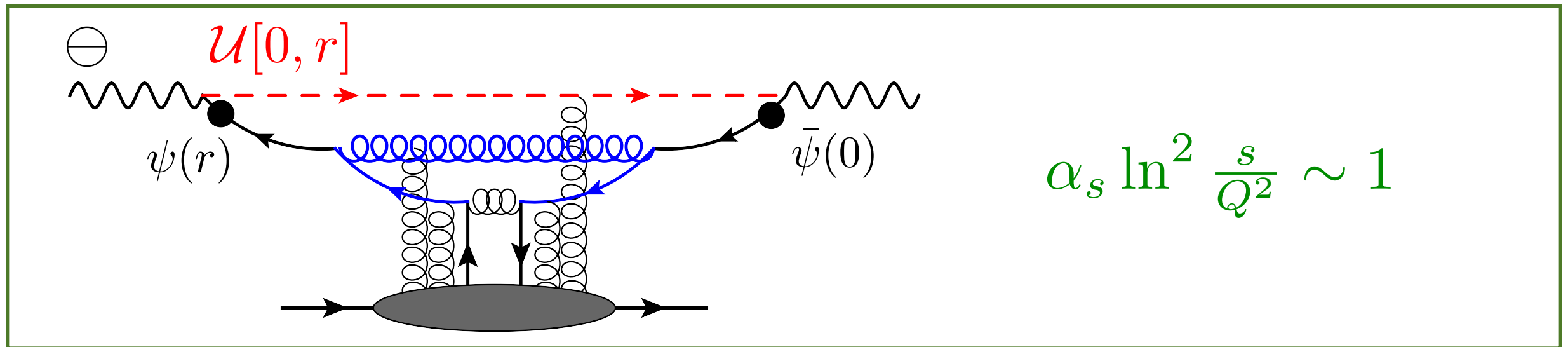
RBRC Workshop: Emerging Spin and
Transverse Momentum Effects in pp / pA

Small-x Helicity Evolution



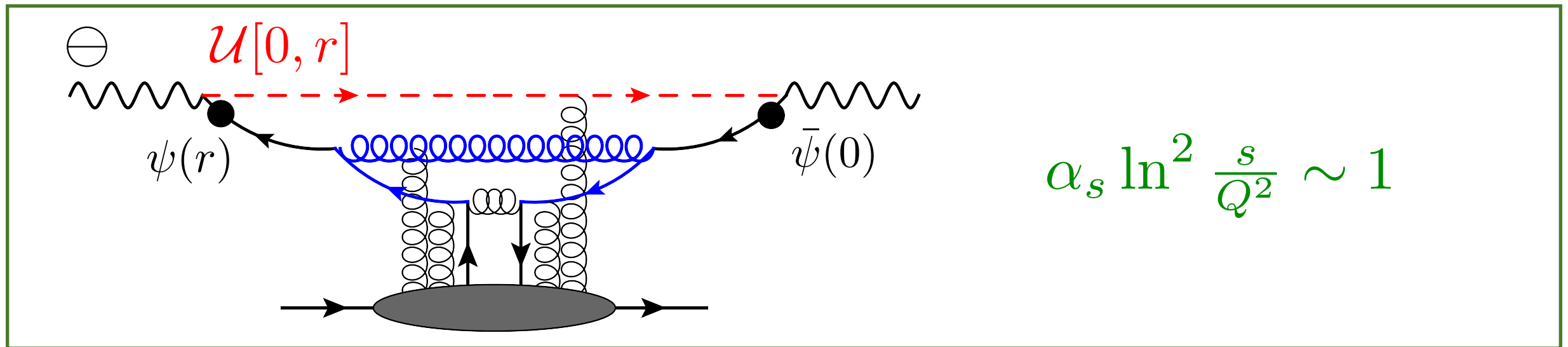
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- **Quark helicity** at very **small x** evolves by the radiation of **soft polarized quarks** and **gluons**.
- We can formulate a **small-x evolution** equation for the quark helicity, which appears to show **rapid growth at small x**.
- But helicity evolution is **much more complex** than unpolarized small-x evolution....

Motivation: Proton Spin Puzzle

- The “Proton Spin Budget” is described by the Jaffe-Manohar Sum Rule.

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_q + L_g$$

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➡ **Quark spins** from polarized DIS

➡ **Gluon spins** from in polarized proton-proton collisions

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$$0.001 < x < 1$$

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- **Proton structure is much more complex** than previously believed!

➡ Orbital angular momentum?

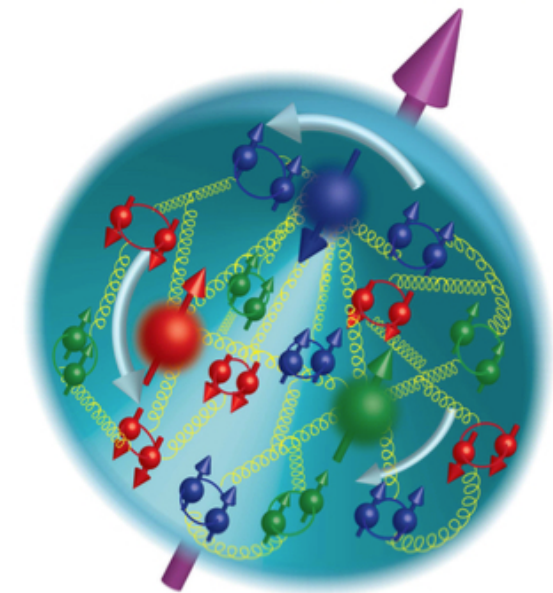
➡ **Polarization at very small x ?**

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Definition: TMD Parton Distributions

$$\underline{\phi_{\alpha\beta}(x, \vec{k}_{\perp})} = \int \frac{d^2-r}{(2\pi)^3} e^{ik \cdot r} \langle h(p, S) | \bar{\psi}_{\beta}(0) \mathcal{U}[0, r] \psi_{\alpha}(r) | h(p, S) \rangle$$



Transverse Momentum Dependent
Parton Distribution Functions

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		Γ	γ^+	$\gamma^+ \gamma^5$	$\gamma^+ \gamma_{\perp}^i \gamma^5$
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			Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U		$f_1 = \odot$		$h_1^{\perp} = \odot - \odot$ Boer-Mulders
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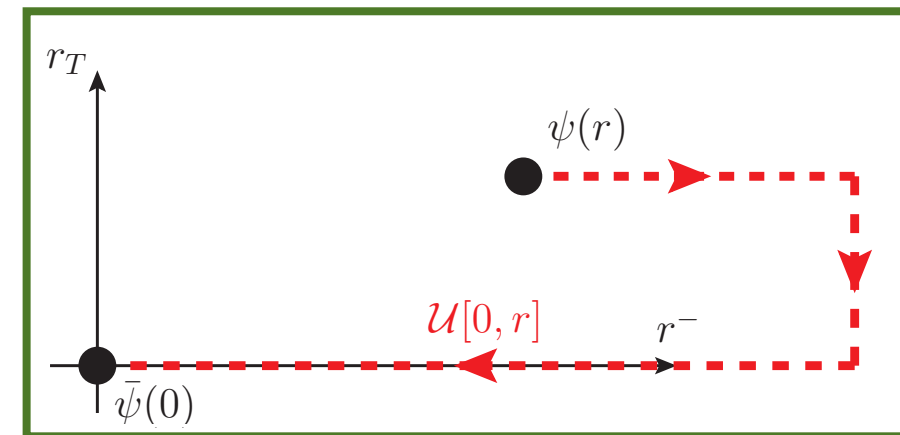
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Staple-shaped Gauge Link
encodes final-state interactions

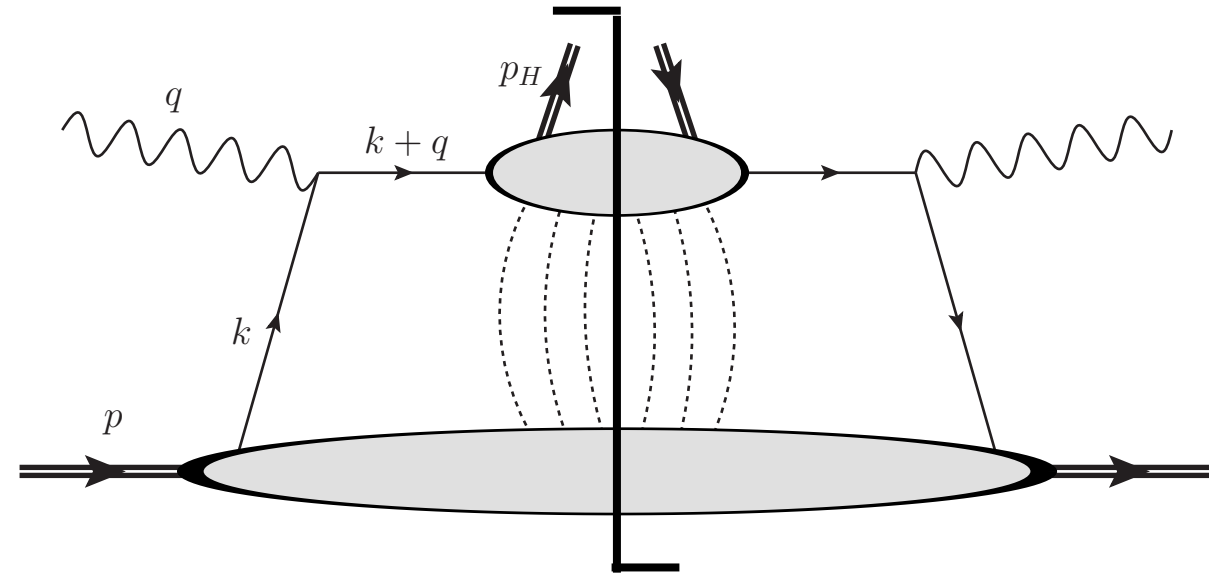
TMD's at Large x

Semi-Inclusive
Deep Inelastic Scattering (SIDIS)

$$e + p \rightarrow e' + h + X$$

Large- x Kinematics:

$$\hat{s} \sim Q^2 \gg k_T^2$$
$$x = \frac{Q^2}{\hat{s} + Q^2} \sim \mathcal{O}(1)$$



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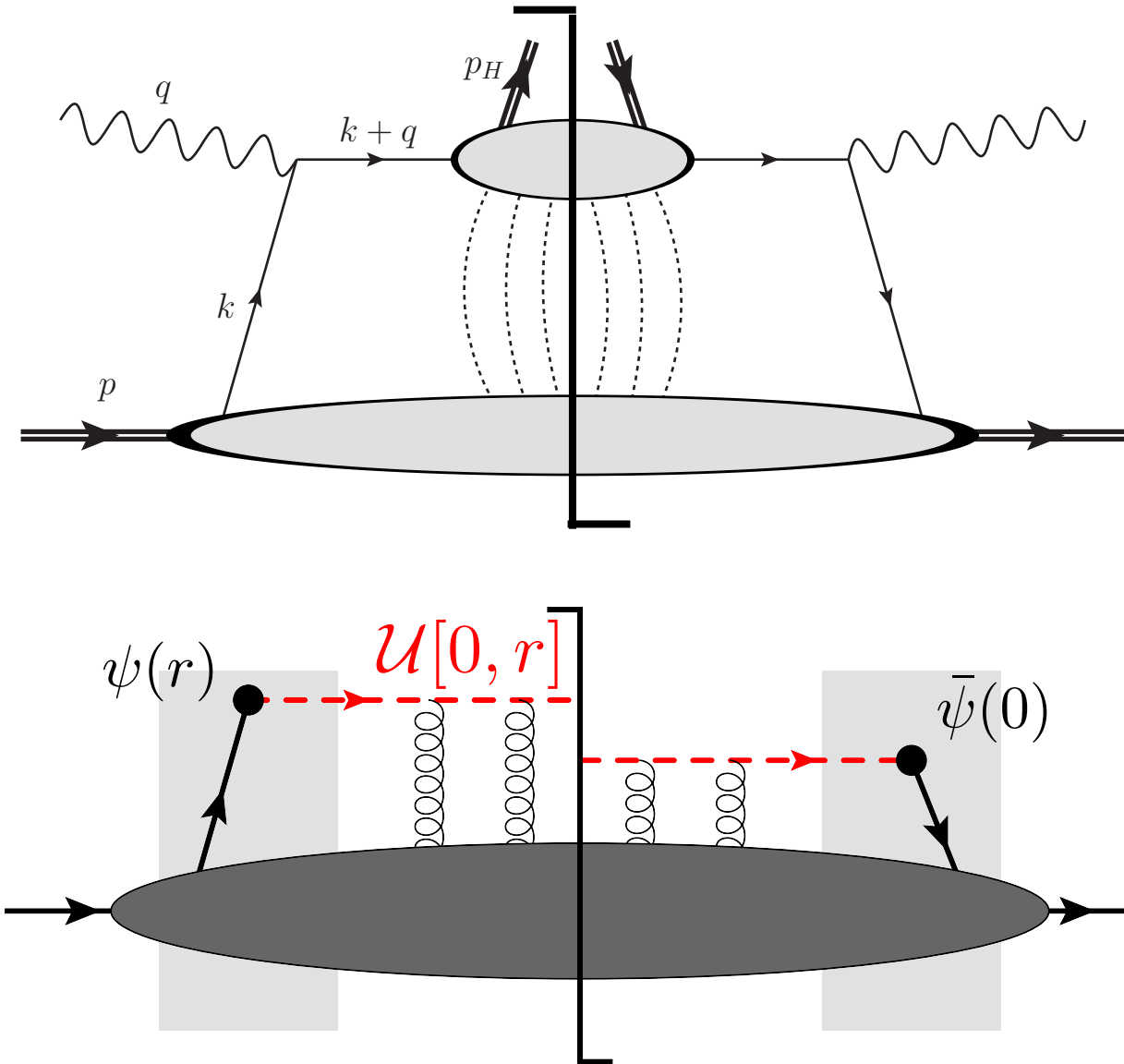
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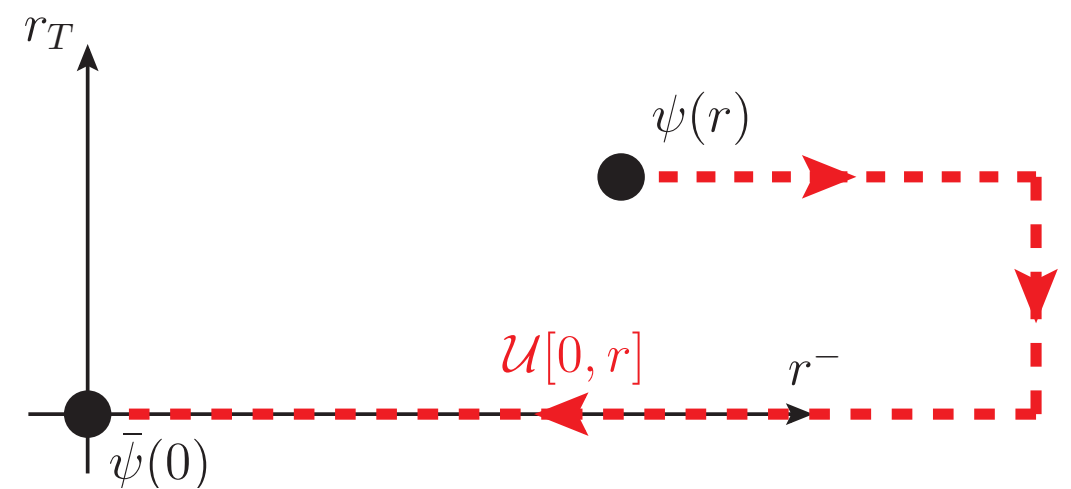
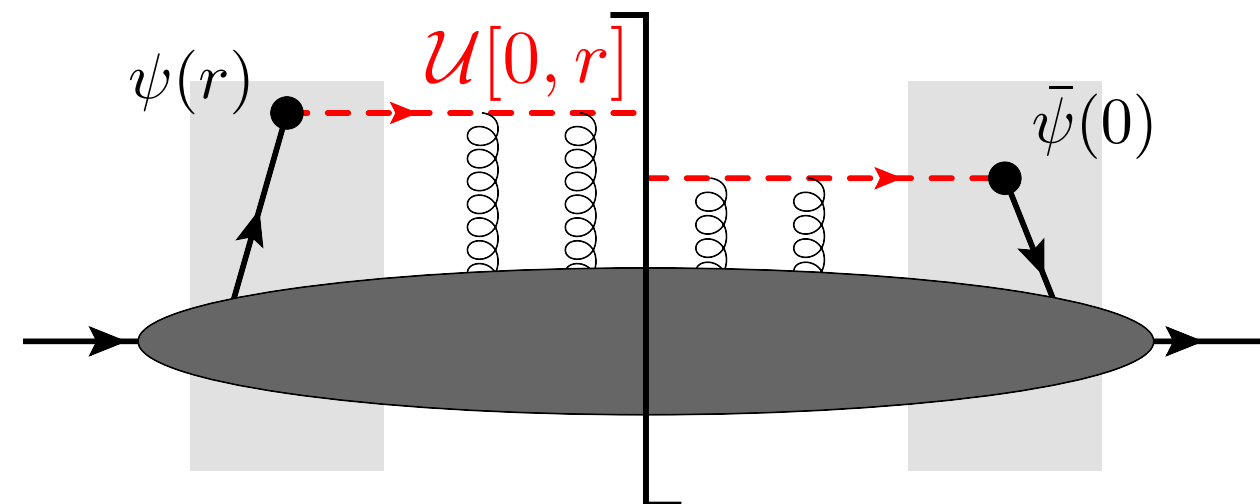
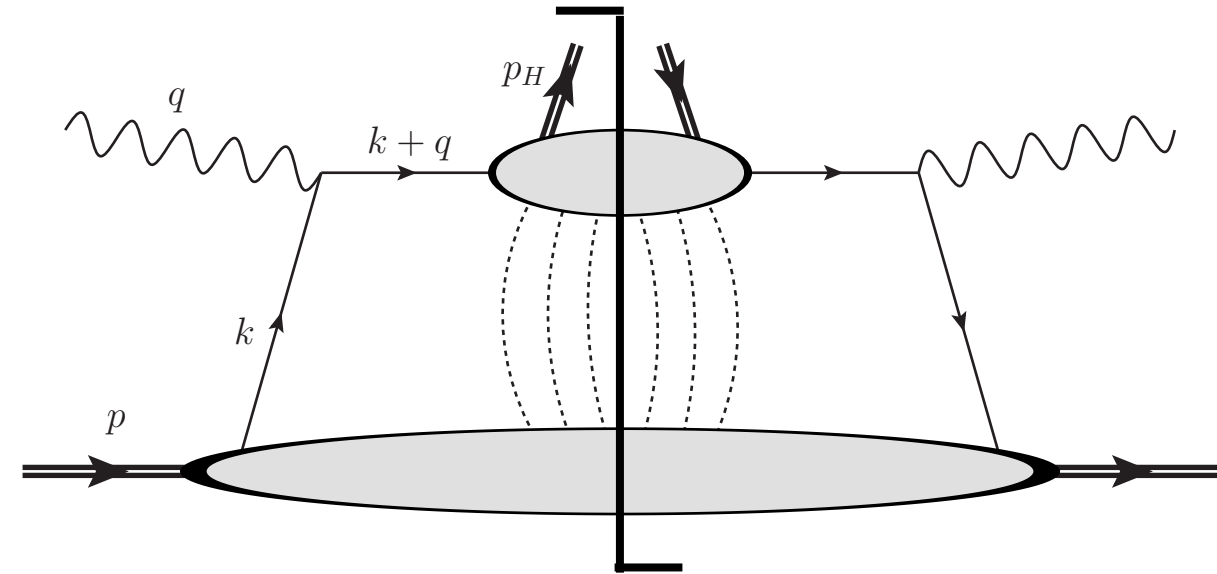
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- **Staple-shaped gauge link** encodes final-state interactions



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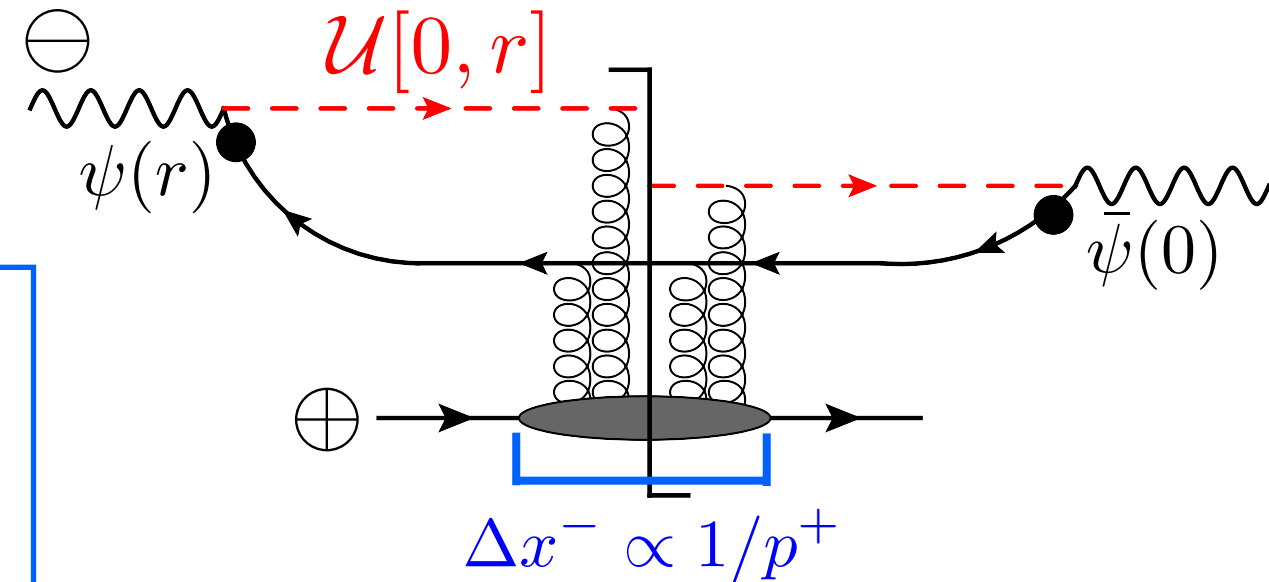
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$$\Delta t < \frac{1}{m_N x}$$

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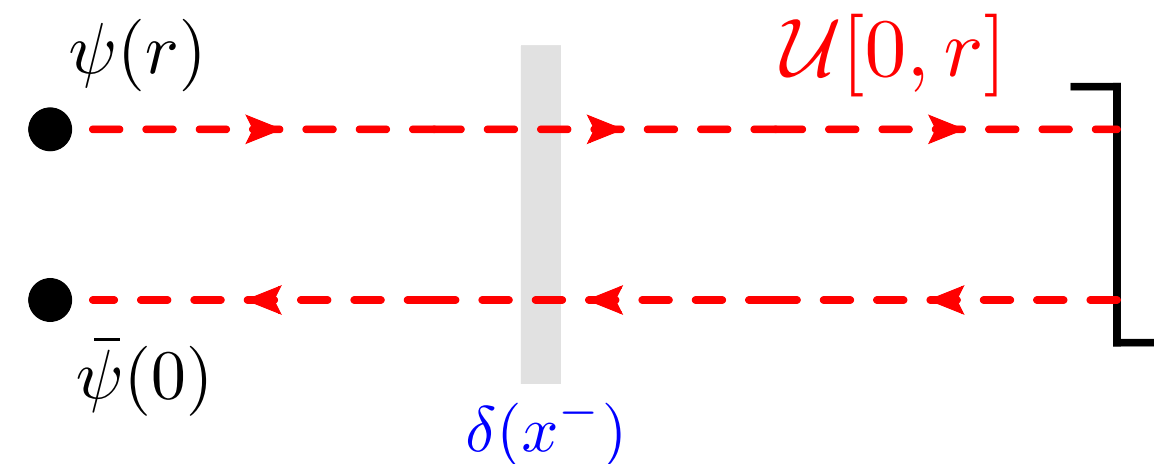
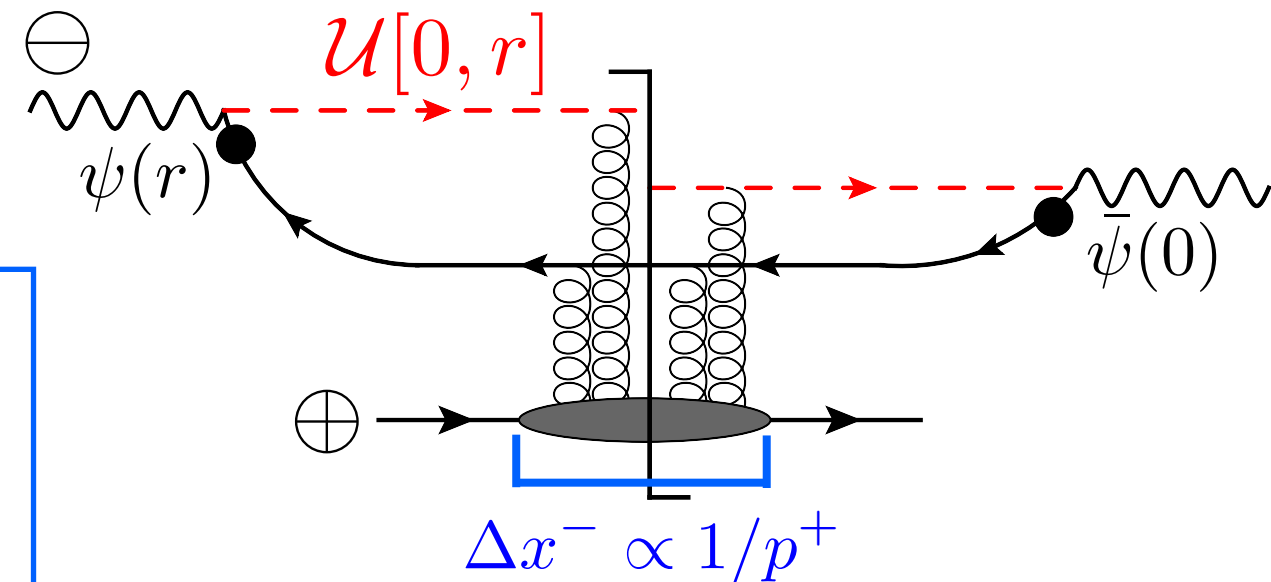
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- Proton is Lorentz-contracted to a “shockwave”.

➡ Gauge link covers the entire proton.

➡ Infinite dipole degrees of freedom at small x



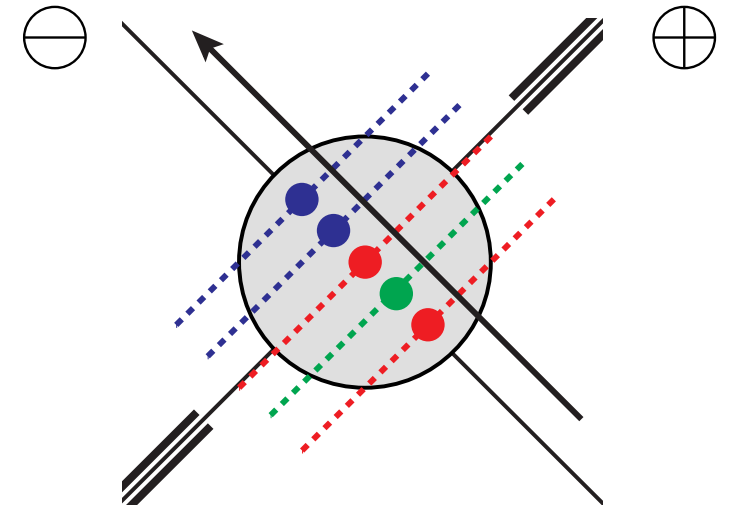
$$S_{xy} = \frac{1}{N_c} \text{Tr} [V_x V_y^\dagger]$$

Small-x Initial Conditions: Classical Gluon Fields

- Long-lived projectile sees **whole target coherently**.
- ➔ **High gluon density at small x enhances multiple scattering**

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Y. Kovchegov
2:30 - 3:00



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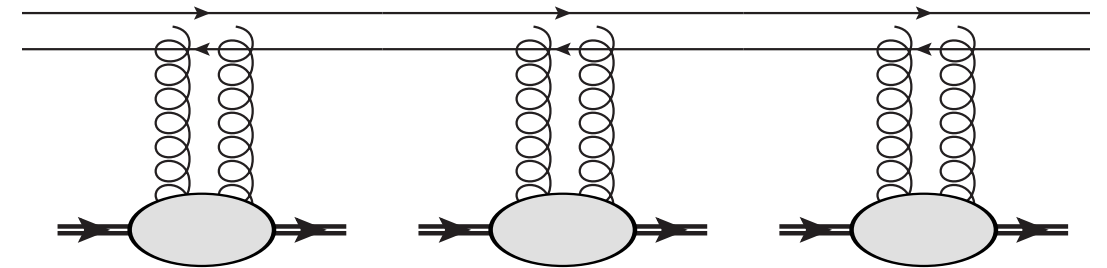
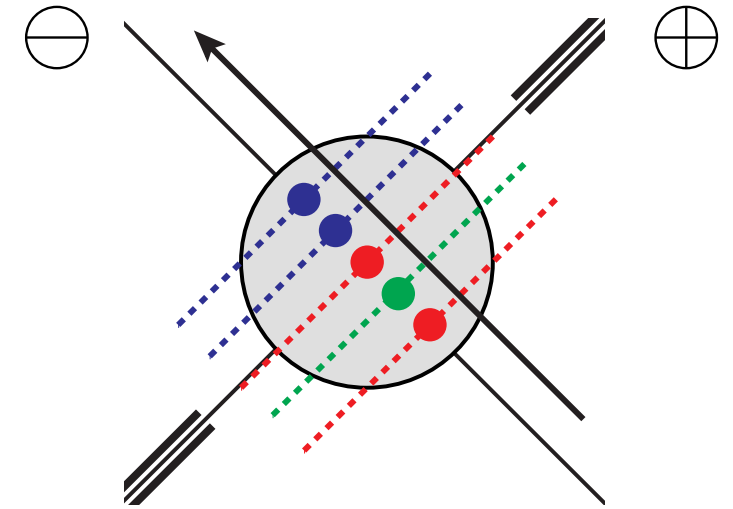
- High density rescattering can be **systematically re-summed**

➔ **Classical gluon fields!**

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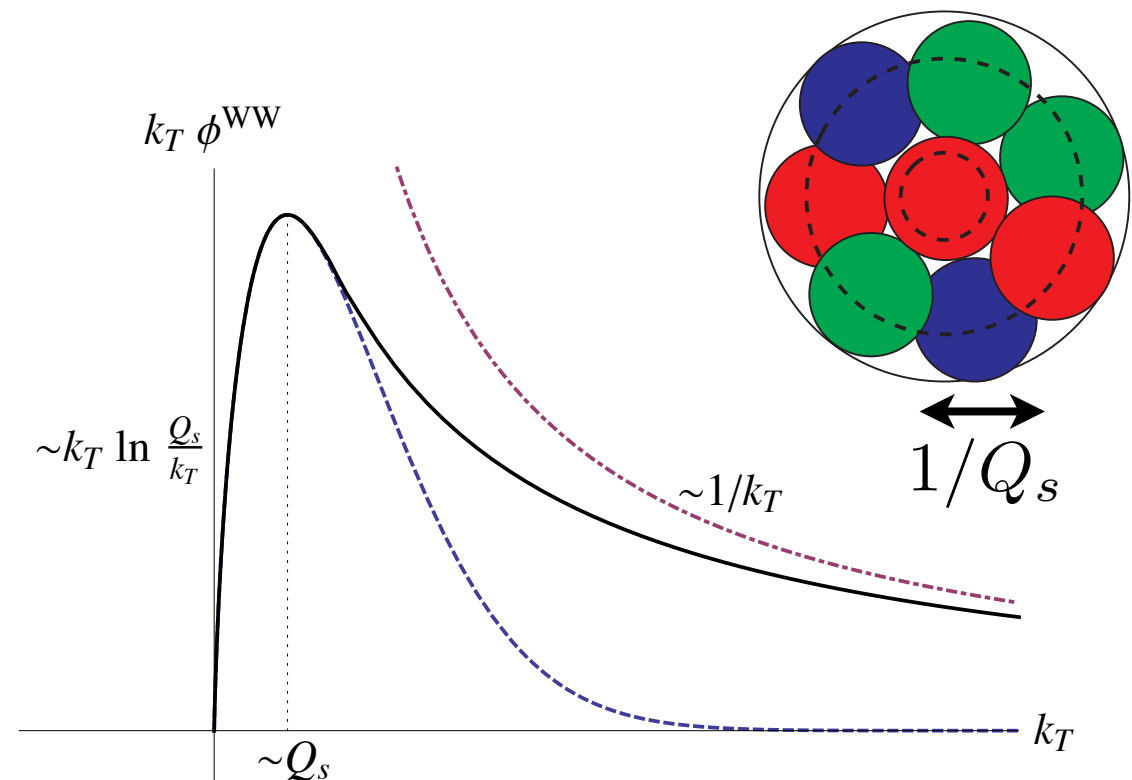
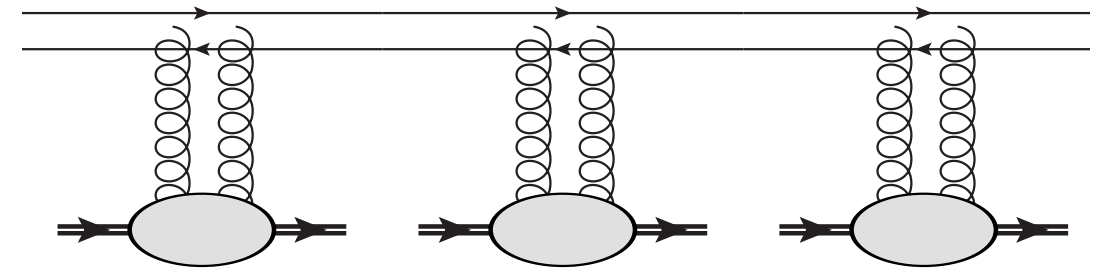
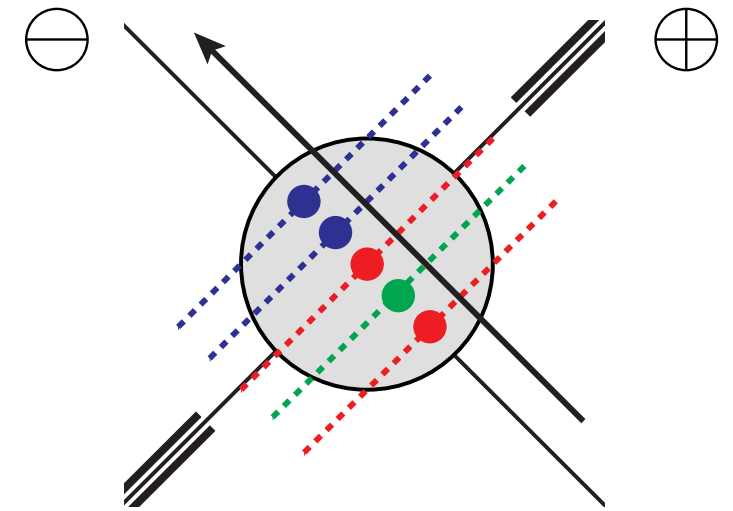
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- Charge density defines a **hard momentum scale** which **screens the IR gluon field**.

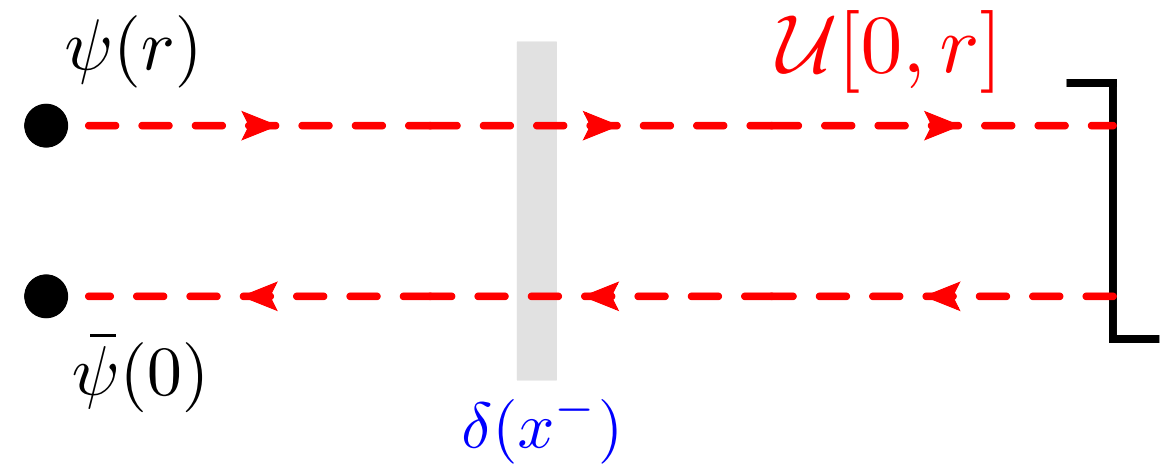
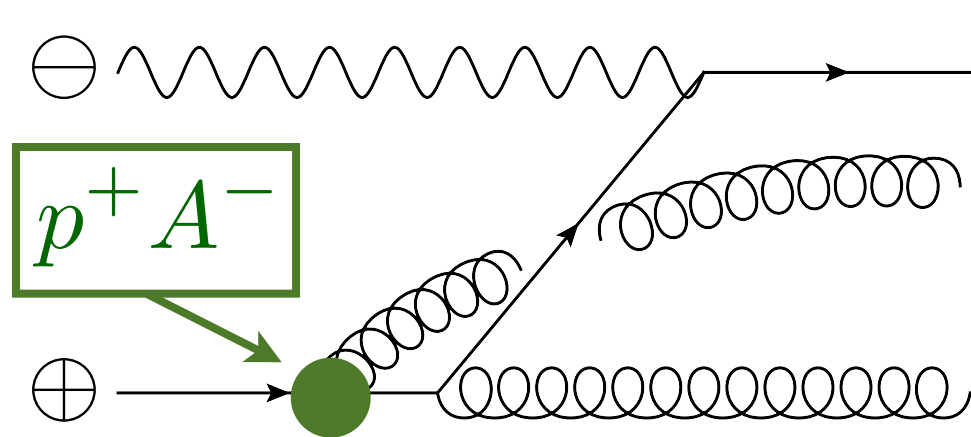
Both: $Q_s^2 \propto \alpha_s^2 A^{1/3} \propto \alpha_s \rho$
 $Q_s^2 \gg \Lambda^2$

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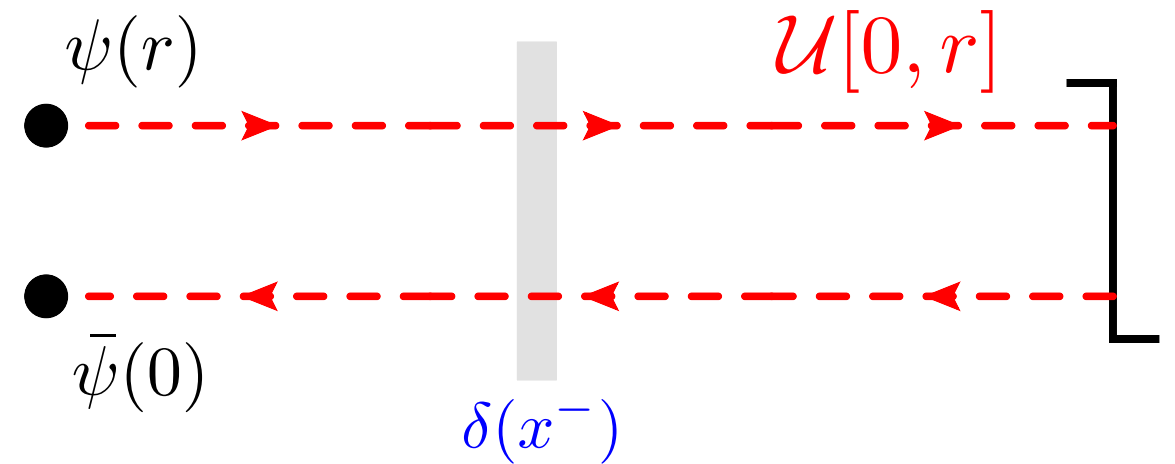
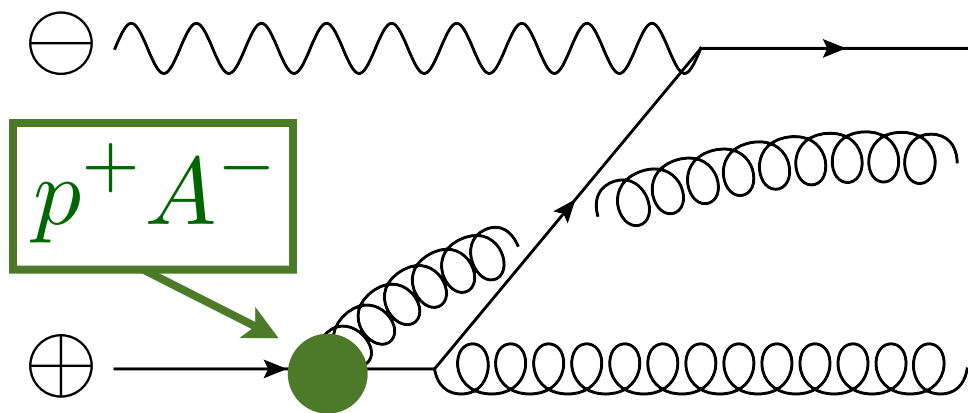


Quantum Evolution in the Light-Cone Gauge



- High-energy radiation from a \oplus moving particle couples to A^-
- ➔ In $A^- = 0$ gauge this radiation is suppressed.

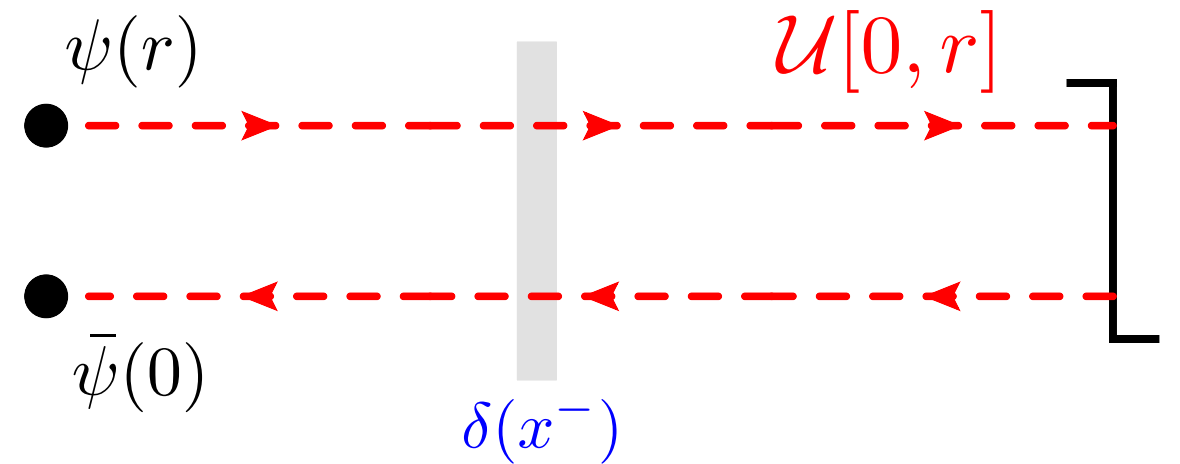
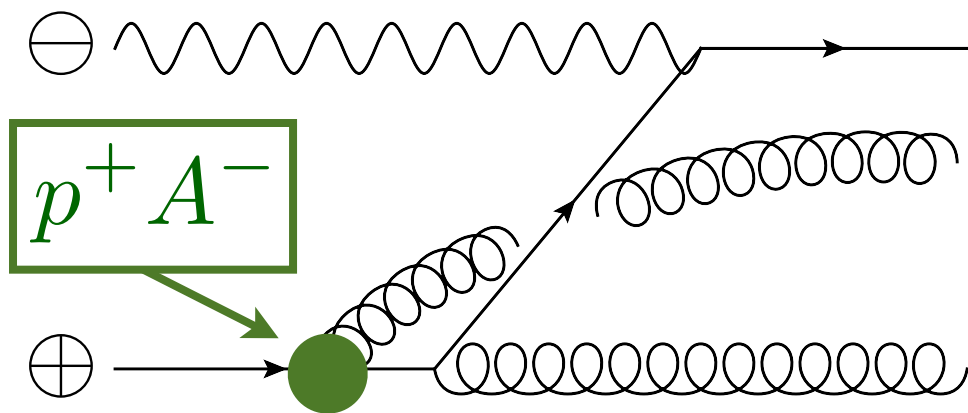
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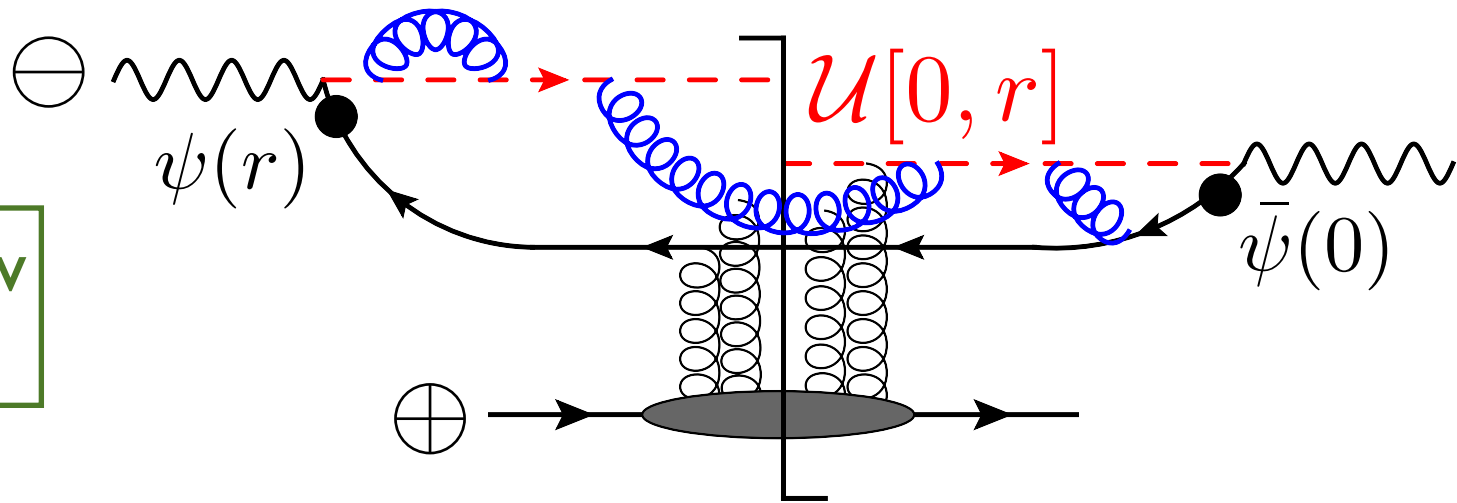


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- For classical fields and leading-log evolution, $A_\perp = 0$ as well.
- ➡ The transverse part of the gauge link does not contribute.

Unpolarized Small-x Evolution



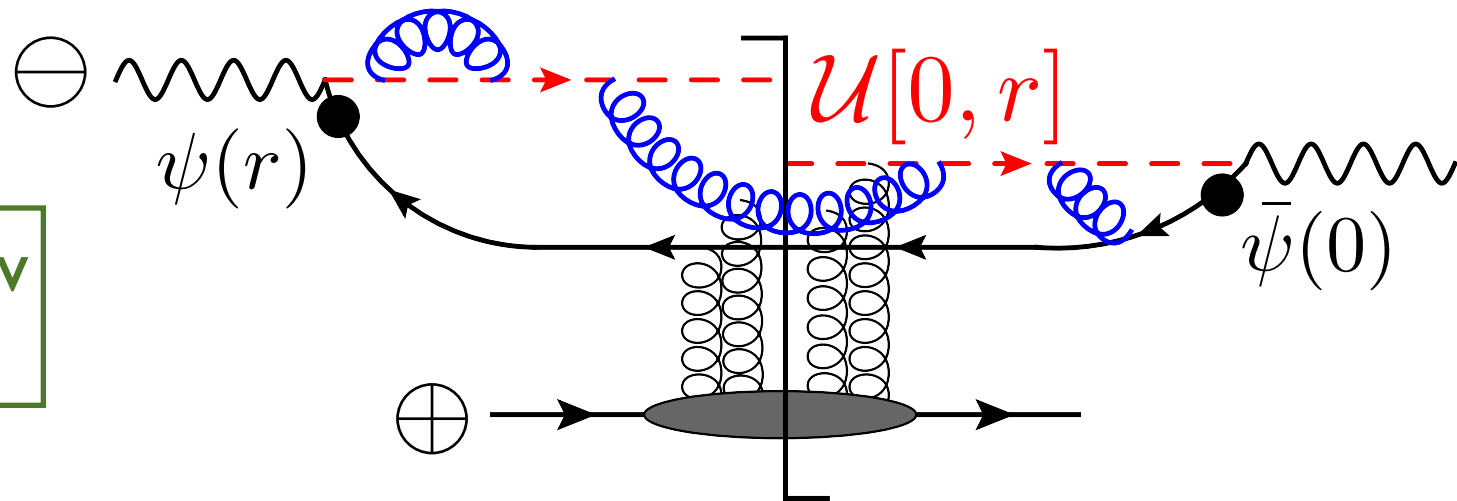
$$S_{xy} = \frac{1}{N_c} \text{Tr} [V_x V_y^\dagger]$$

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- The quark dipole radiates **soft gluons before and after scattering**.
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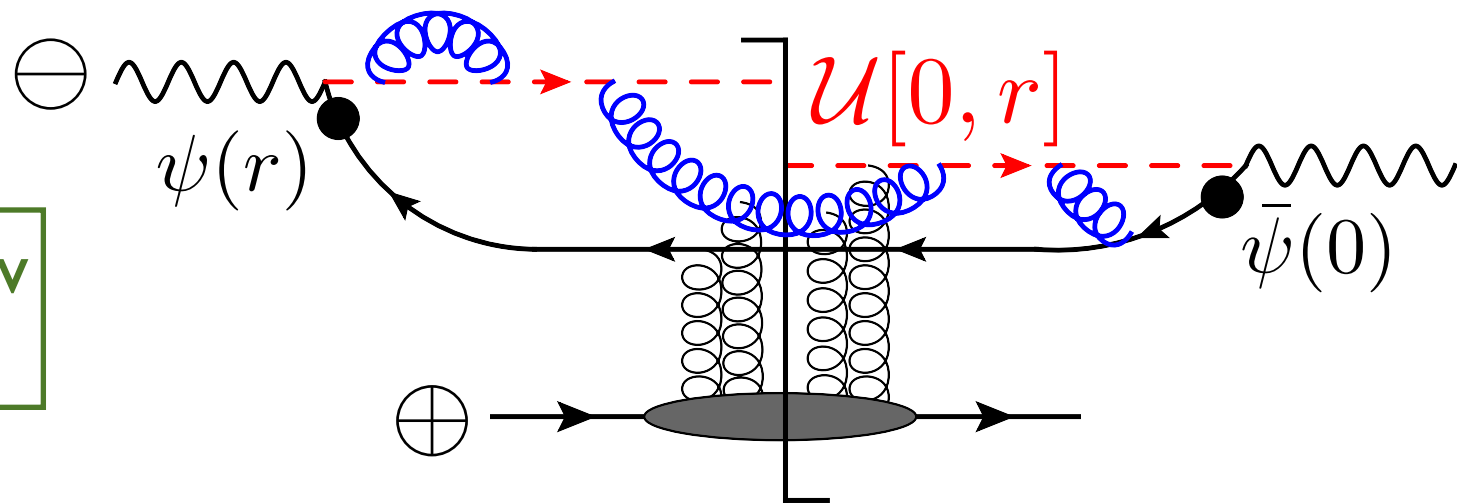
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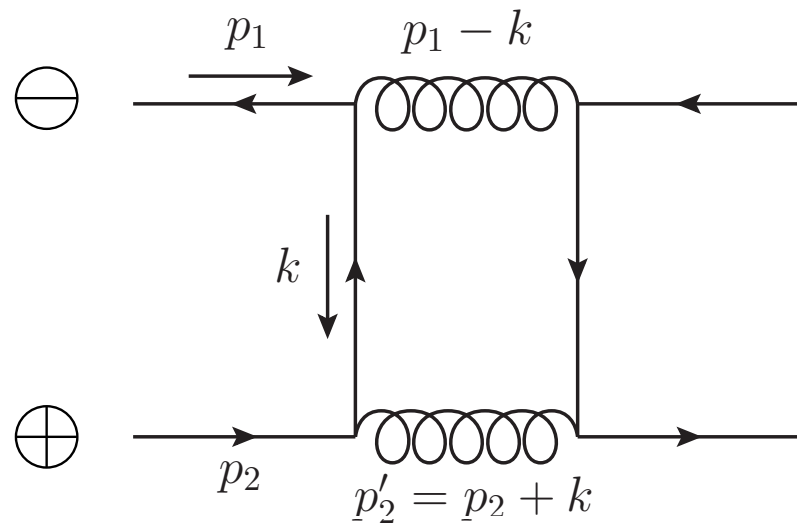
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- Evolution closes in the large N_c limit (BK eqn.) $Q_s^2(x) \sim \left(\frac{1}{x}\right)^{0.3}$

Leading-Order Spin Dependence

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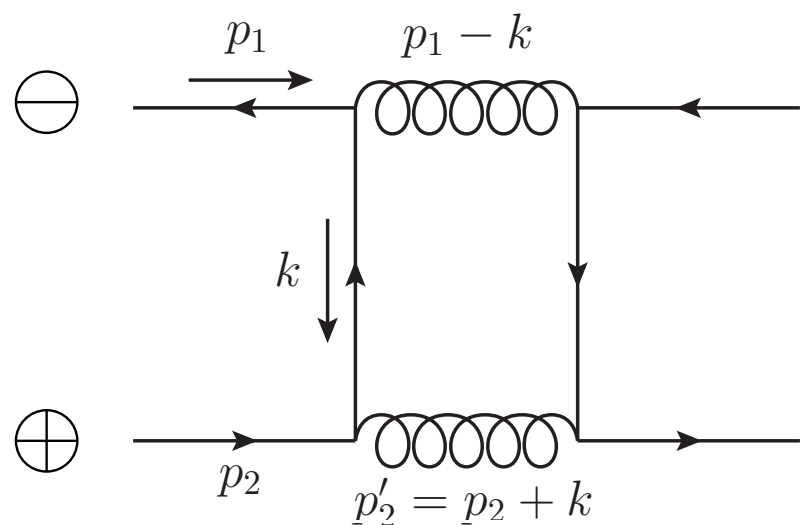
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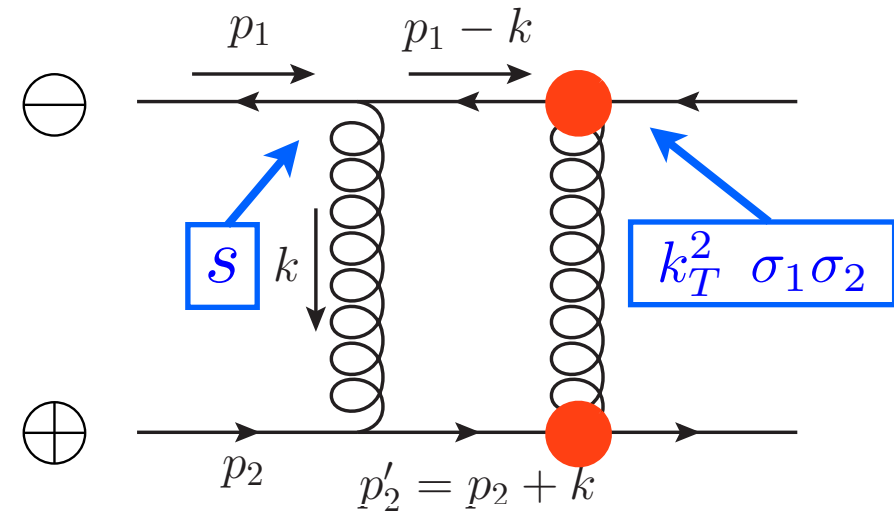
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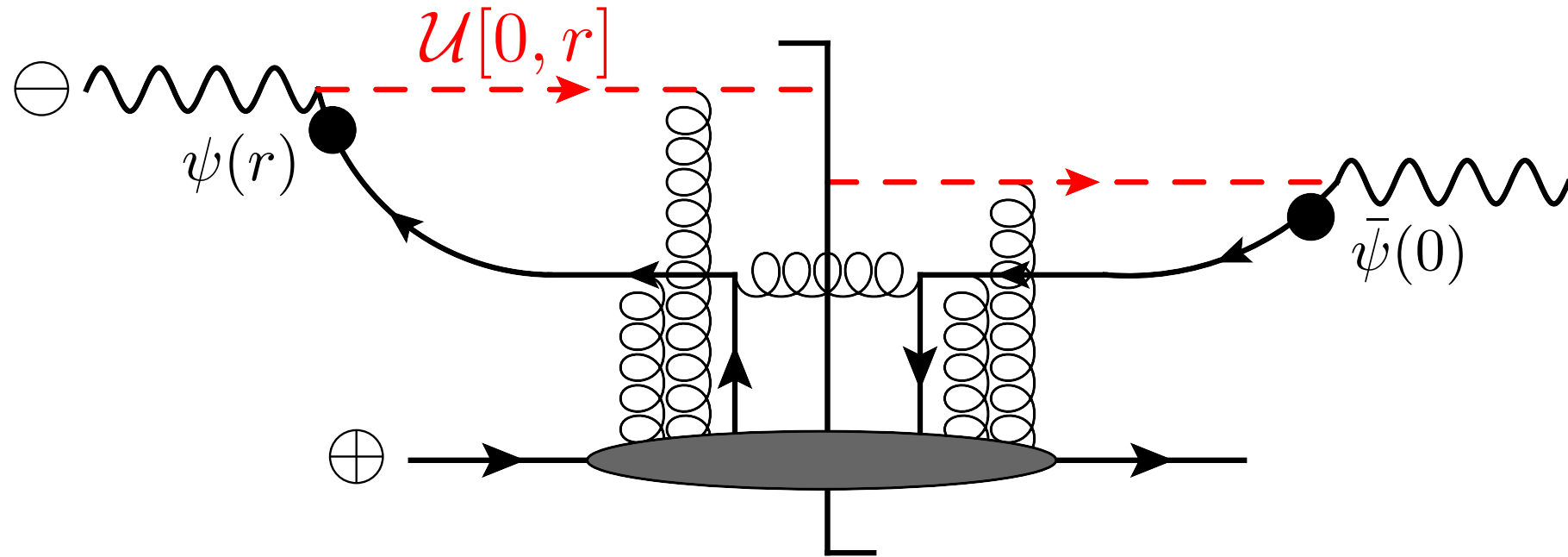
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- Sub-leading gluon exchange can also transfer spin dependence.

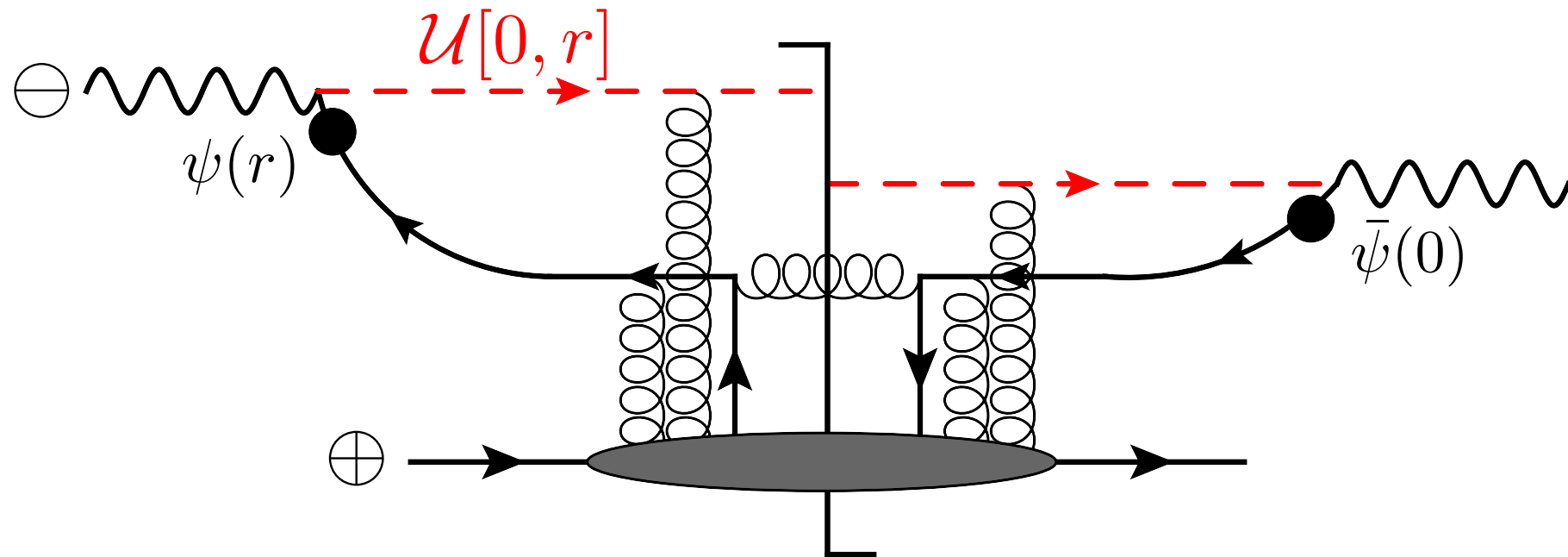
➡ Gluon exchange can mix with quark exchange.

Spin-Dependent Initial Conditions



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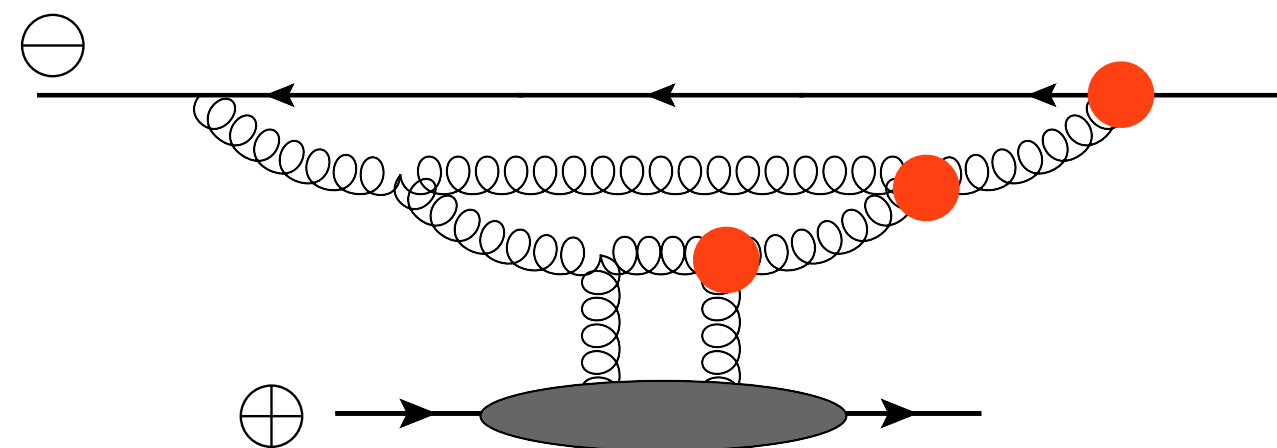
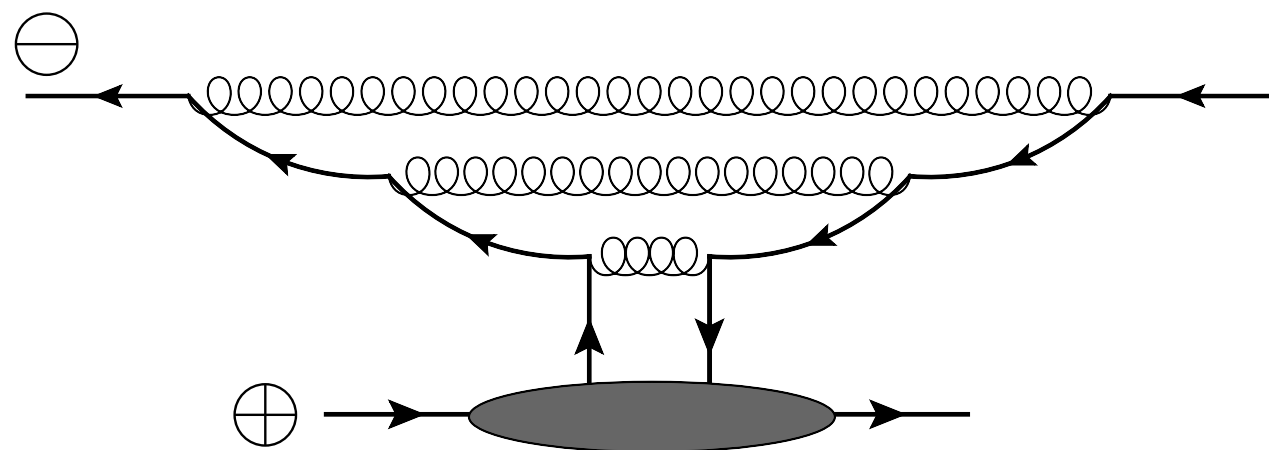


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- “Polarized Dipole Amplitude”:
 - ➡ Quark (gauge link) scatters by an unpolarized Wilson line.
 - ➡ Fermion (antiquark) scatters by a polarized Wilson line.

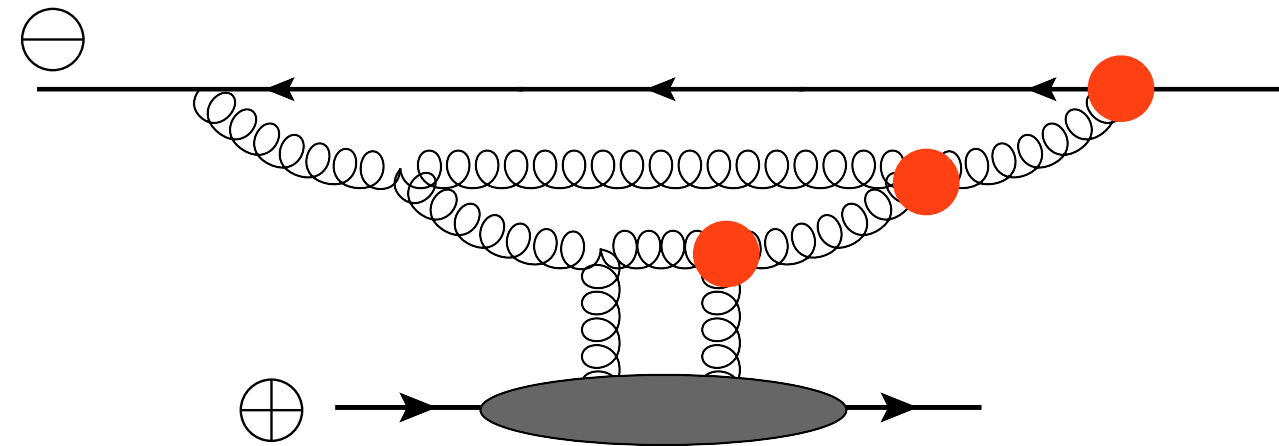
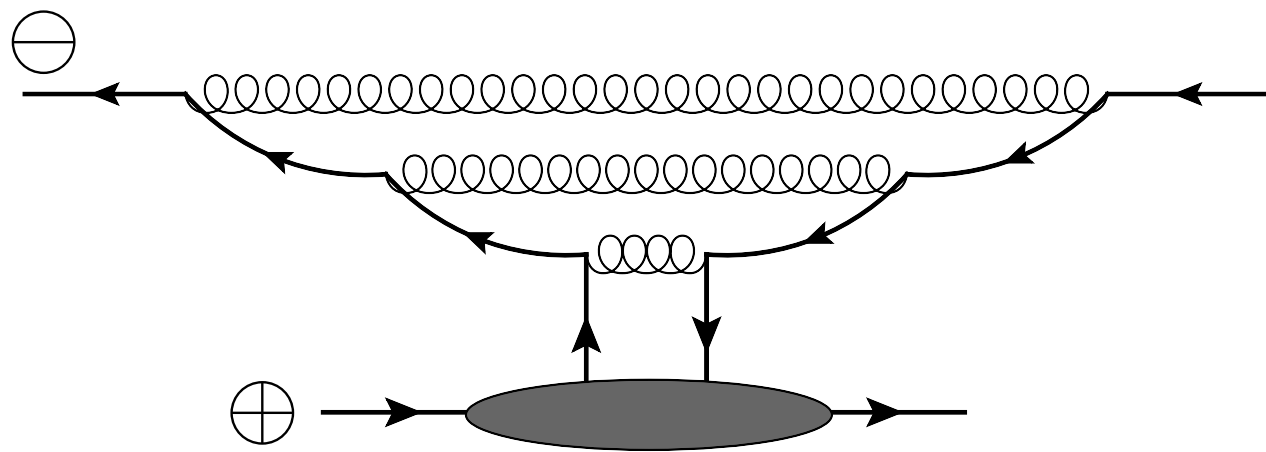
$$G_{xy} \equiv \frac{1}{2N_c} \text{Tr} [V_x V_y^\dagger(\sigma) + V_y(\sigma) V_x^\dagger]$$

Constructing Polarized Splitting Kernels



- **Kernels:** Spin-dependent quark / gluon wave functions
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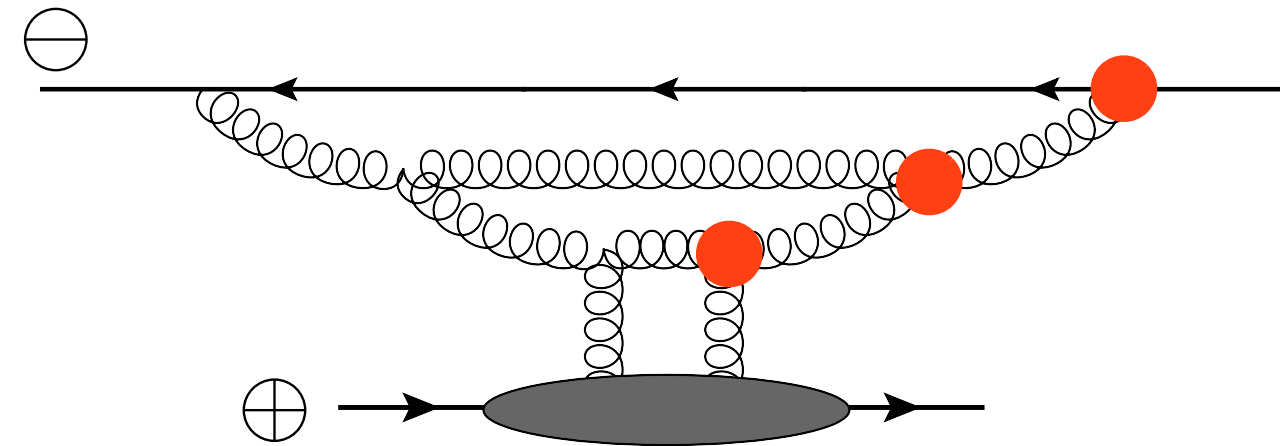
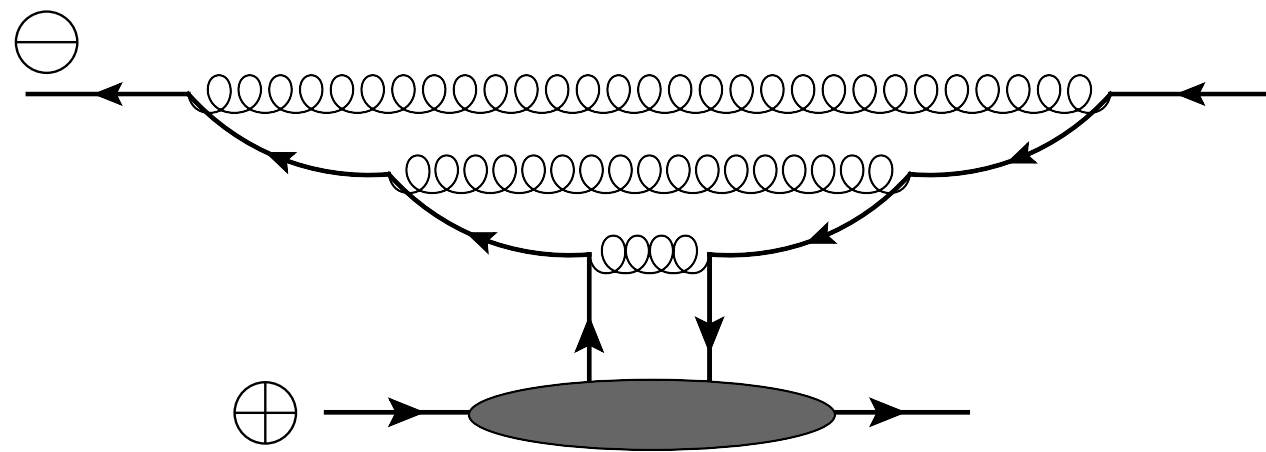
- Requires **longitudinal** and **transverse** momentum ordering

$$1 \gg z_1 \gg z_2 \gg \dots \gg \frac{Q^2}{s}$$

$$Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \dots$$

- ➔ Includes **“infrared”** phase space: $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

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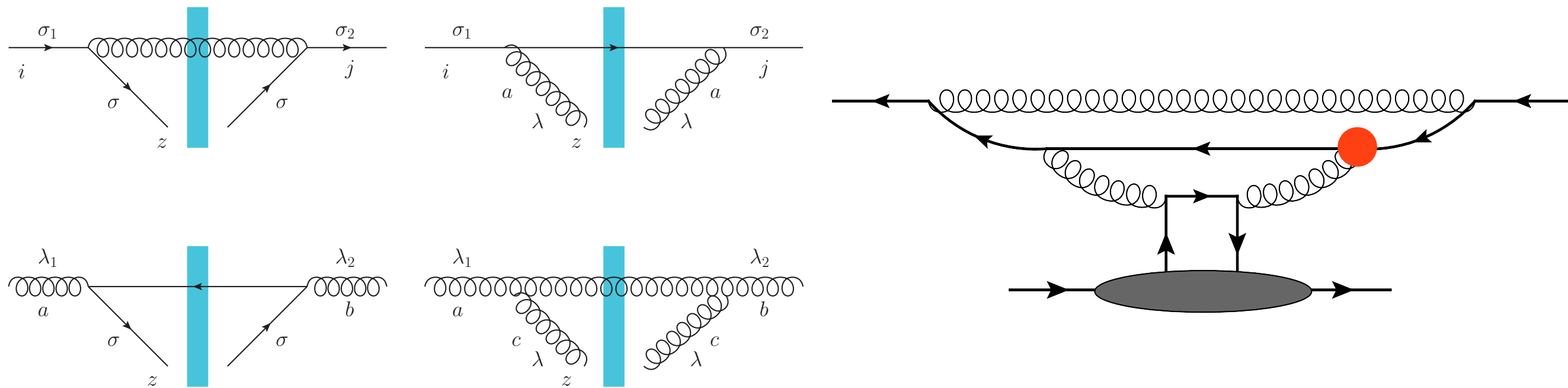
- ➔ Includes **“infrared” phase space:** $k_{1T}^2 \gg k_{2T}^2 \gg k_{1T}^2 \frac{z_2}{z_1}$

- Leads to **double-log evolution.**

- ➔ **Faster** evolution than **unpolarized BK!**

$$\alpha_s \ln^2 \frac{1}{x} \sim 1$$

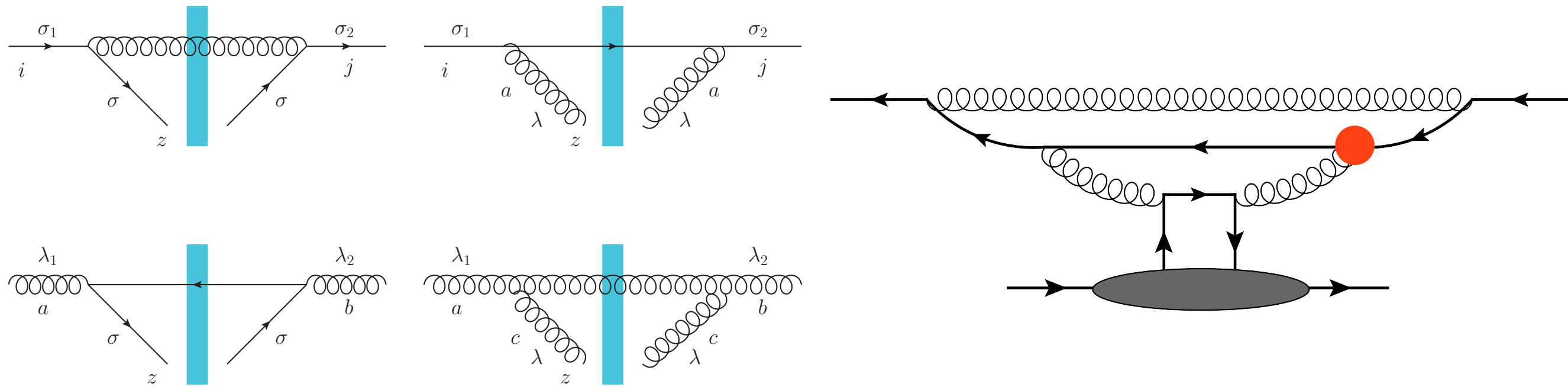
Solution: Ladder Evolution



- To solve, first keep only the kernels without unpolarized rescattering.

$$\frac{\alpha_s}{2\pi} \int \frac{dz}{z} \int \frac{dk_T^2}{k_T^2} \begin{pmatrix} C_F & 2C_F \\ -N_f & 4N_c \end{pmatrix}$$

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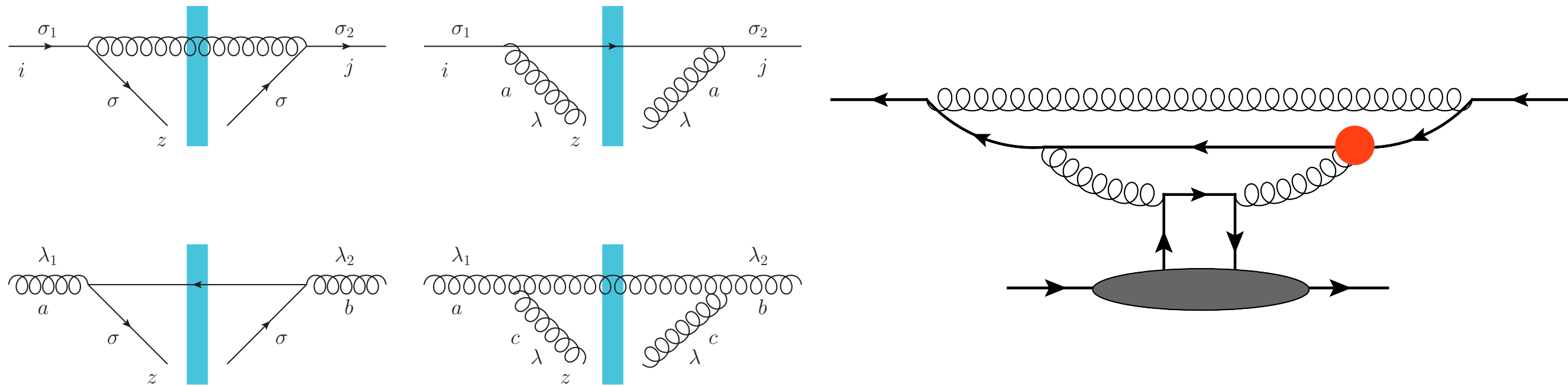
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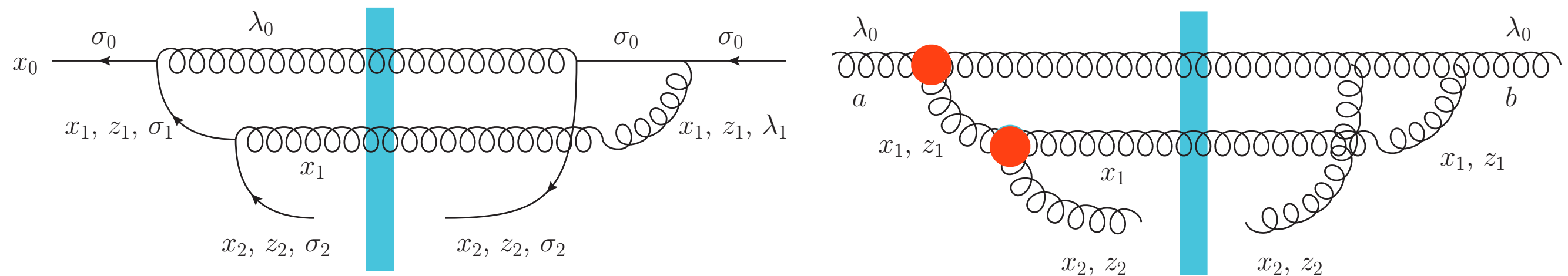
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- Fast growth of quark polarization at small x !
 ➔ Large contribution to the proton spin?

$$S_{xy}(s) \sim \left(\frac{s}{Q^2} \right)^{0.3}$$

The Complication: Non-Ladder Graphs

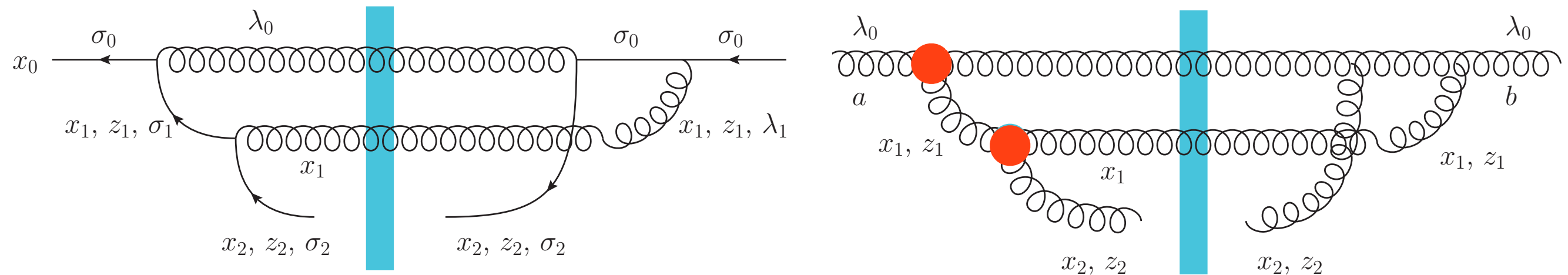


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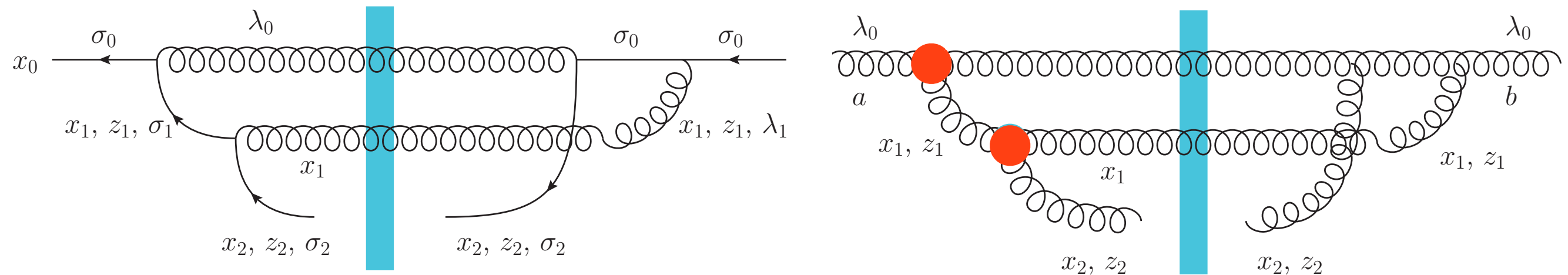
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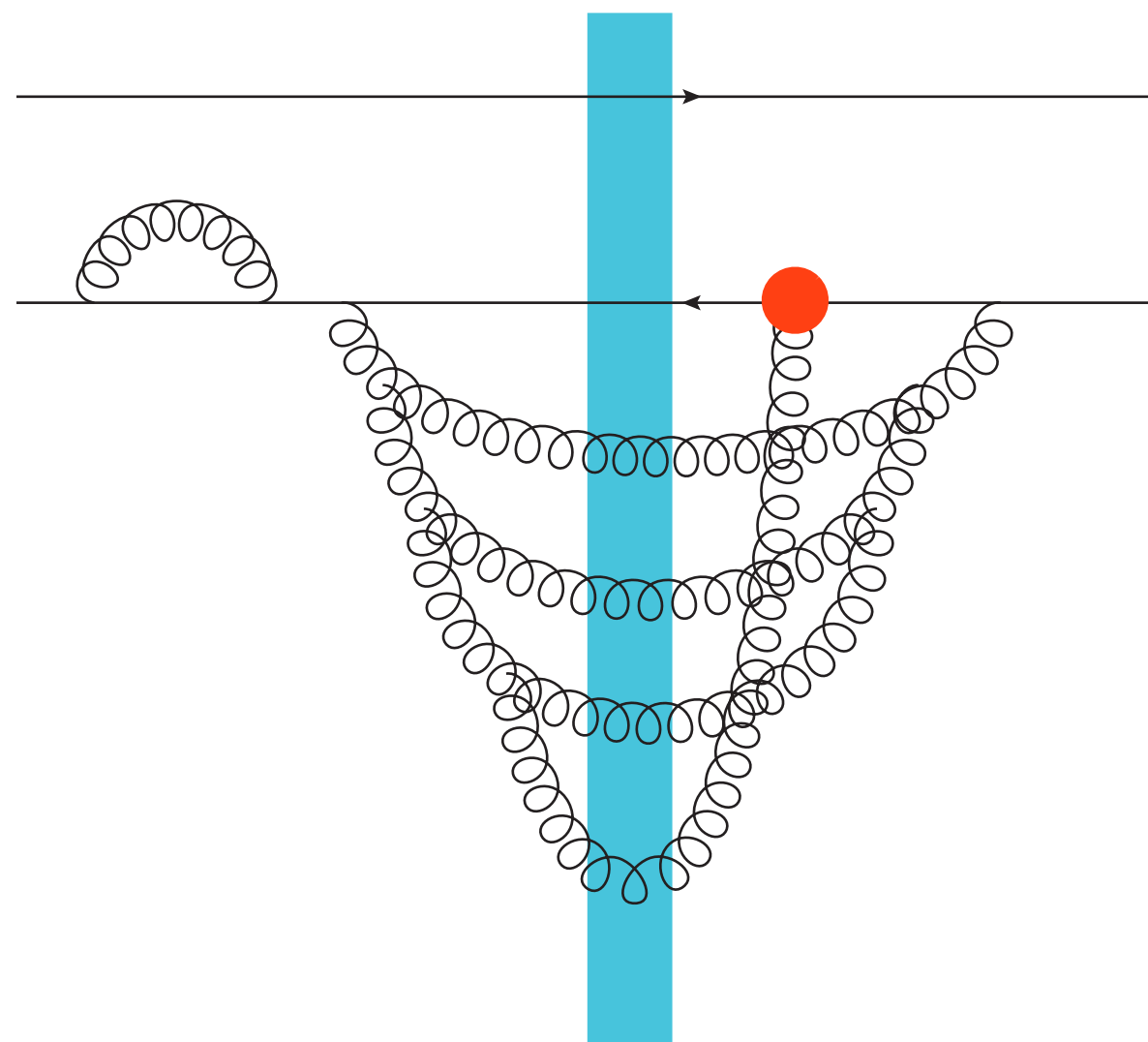
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- **Complication: Gluon non-ladder graphs do not cancel.**

➡ Ladder evolution is an **unjustified truncation**

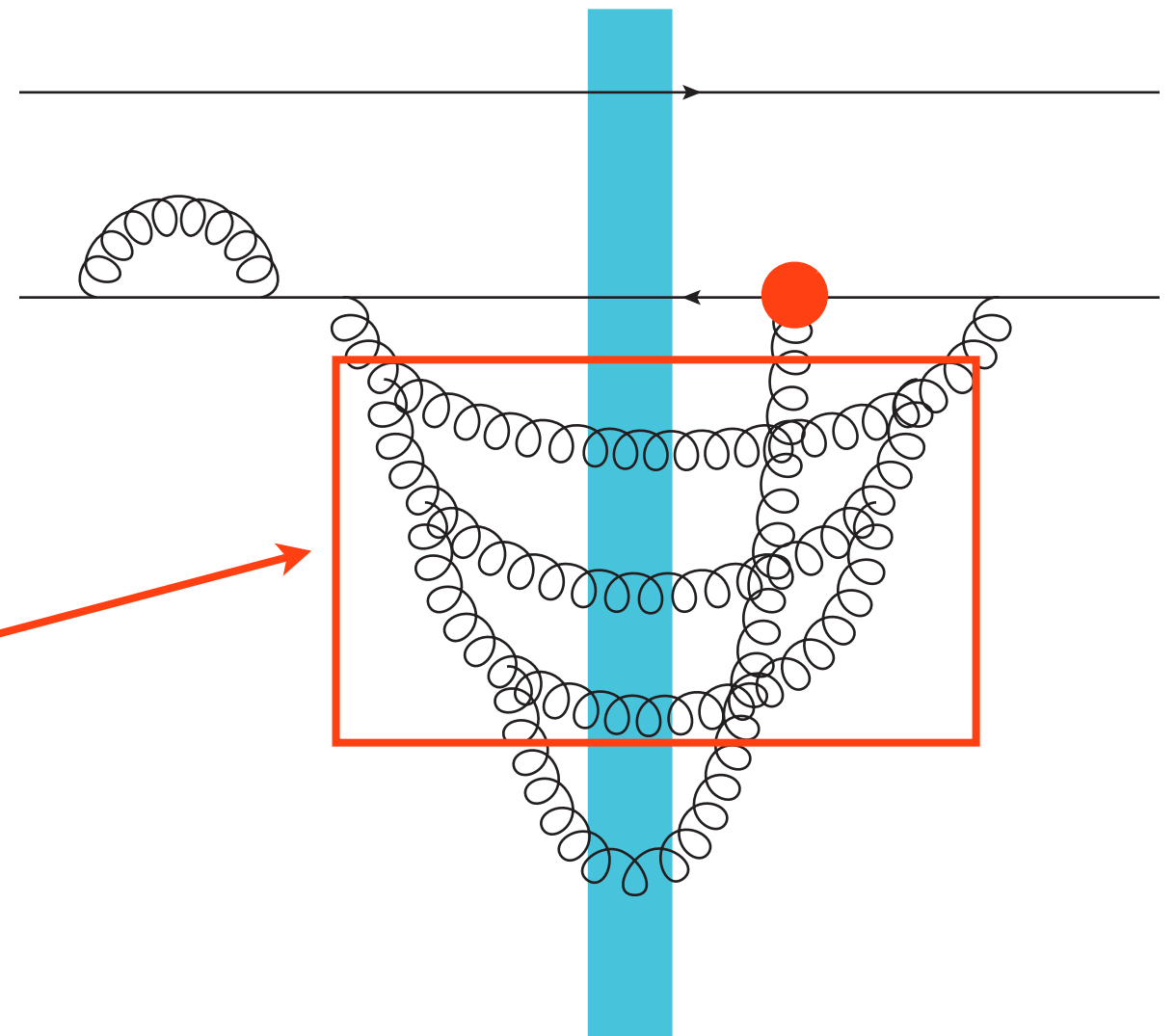
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- Non-ladder gluons can stack in complex ways which still generate leading logarithms.
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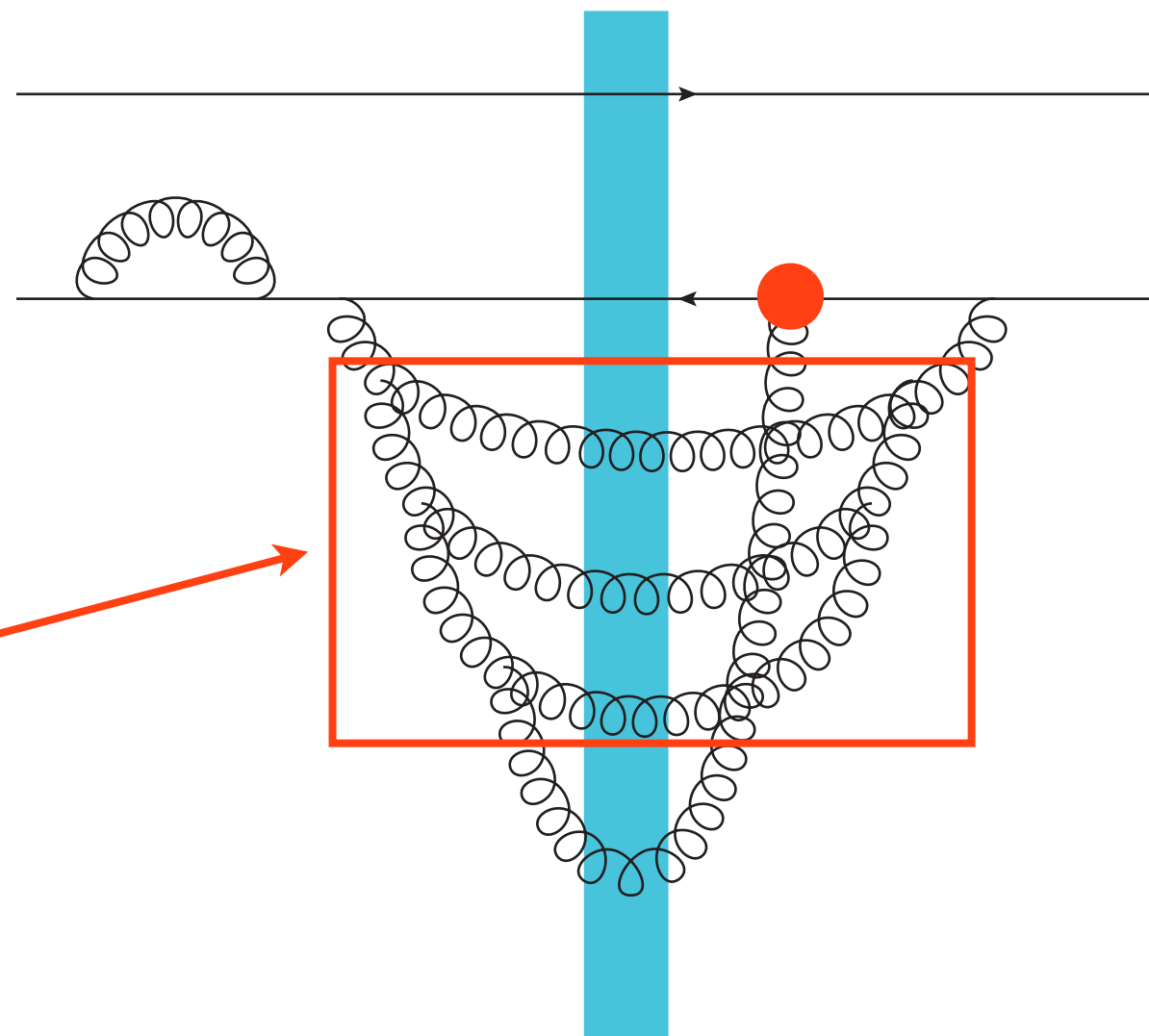
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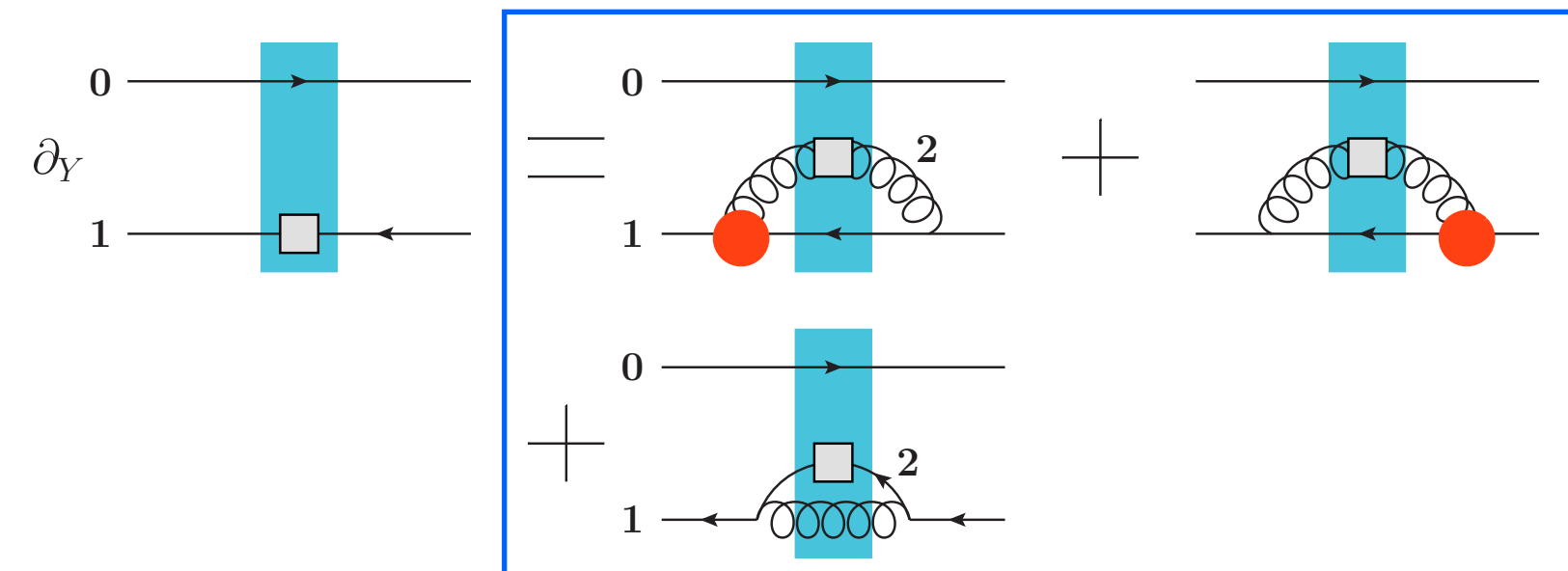
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- ➡ Unpolarized evolution is in a color-octet state (unlike ordinary BK evolution)



Helicity Evolution: Polarized Dipole Operator

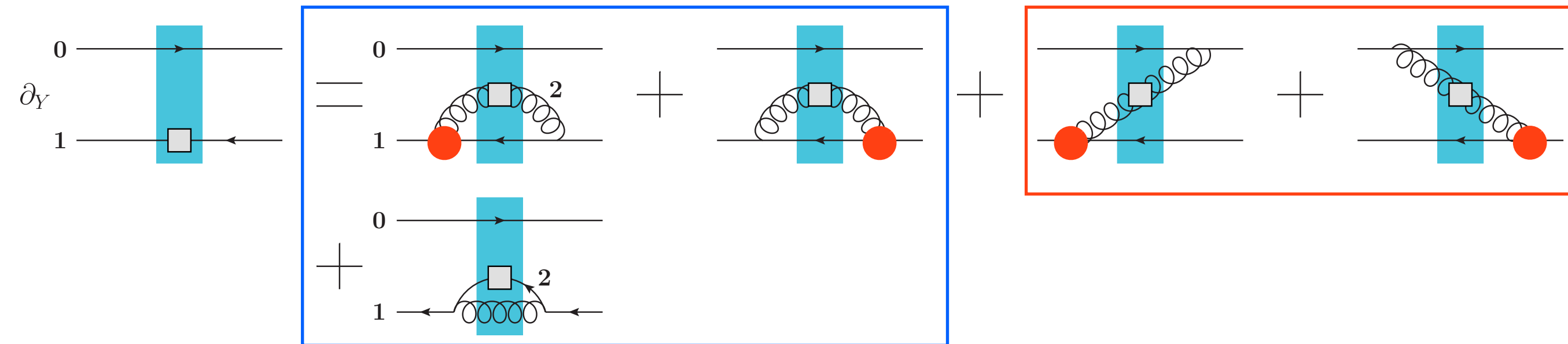
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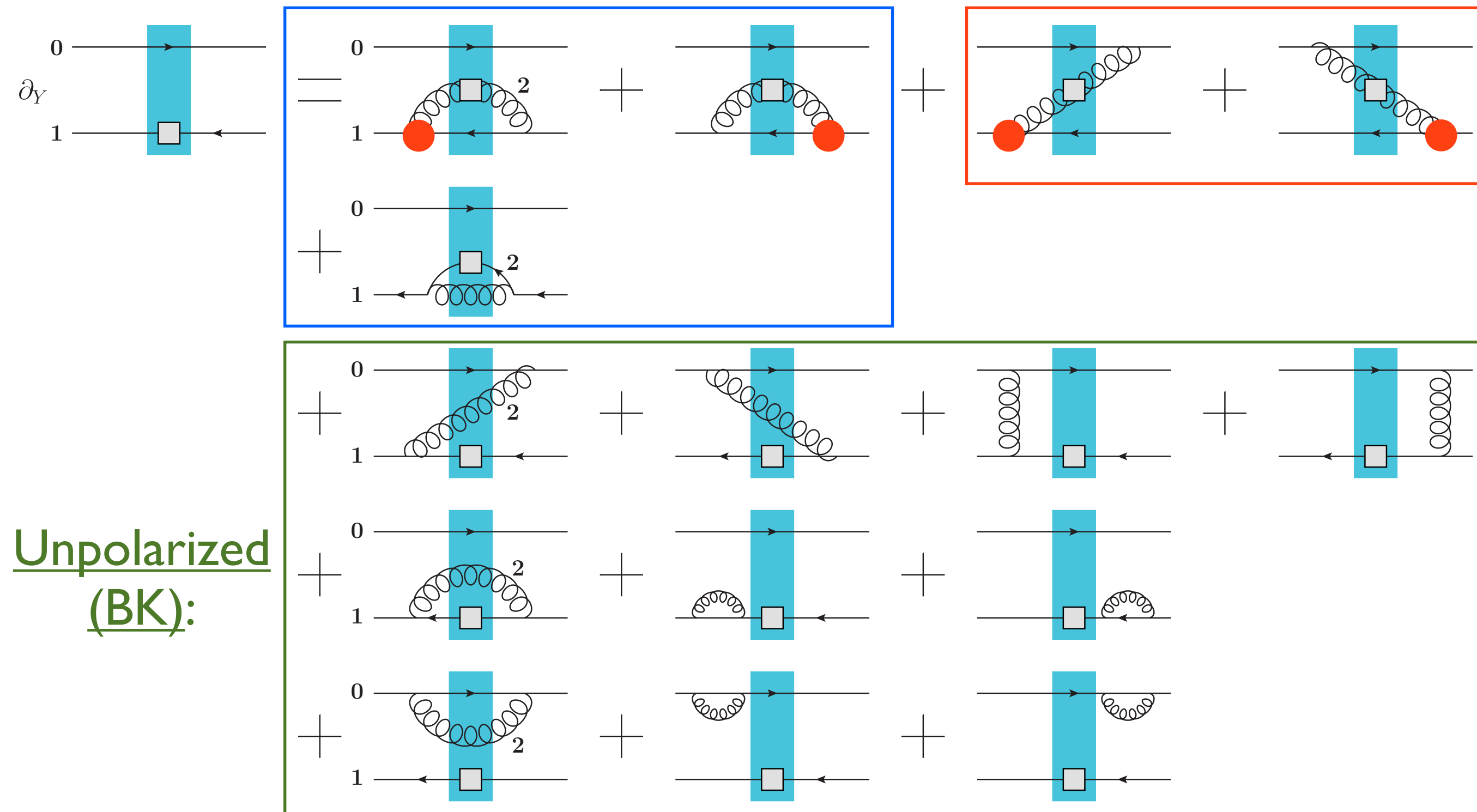
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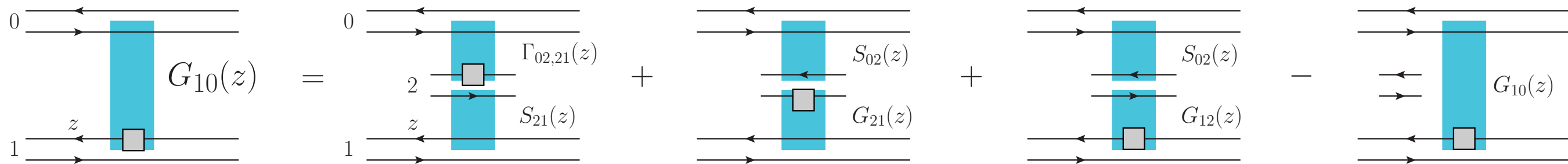
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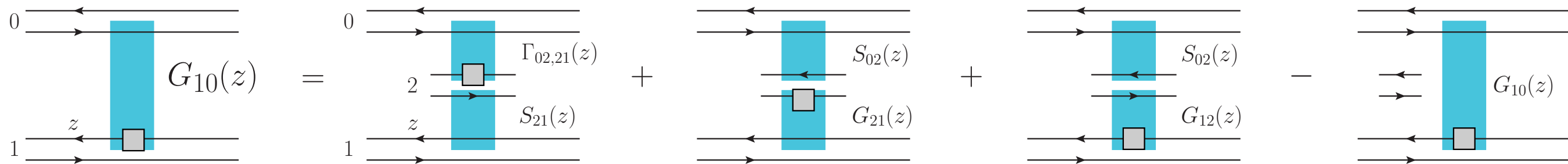


Trying to Solve It: The Large N_c Approximation



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- The **transverse ordering** condition is **not automatically satisfied**.

$$Q^2 \ll \frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \dots$$

- ➡ Polarized dipoles can **depend on their “neighbors”**
- ➡ **More complex** than the large N_c BK equation.

A Better Approximation: Large N_c , N_f

$$\frac{\partial}{\partial \ln z} Q_{10}(z) =$$

$$\frac{\partial}{\partial \ln z} G_{10}(z) =$$

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$$\begin{aligned} \frac{\partial}{\partial \ln z} Q_{10}(z) &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} + \text{Diagram 6} \\ \frac{\partial}{\partial \ln z} G_{10}(z) &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} - \text{Diagram 6} \\ \frac{\partial}{\partial \ln z} A_{10}(z) &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} + \text{Diagram 6} \end{aligned}$$

The diagrams represent various helicity evolution kernels. Each diagram shows a vertical blue bar with a grey square at the bottom. Lines with arrows represent partons. Labels include $\Gamma_{02,21}(z)$, $S_{02}(z)$, $S_{21}(z)$, $G_{21}(z)$, $A_{12}(z)$, $Q_{10}(z)$, $G_{10}(z)$, $A_{10}(z)$, $S_{01}(z)$, $A_{21}(z)$, and $\bar{\Gamma}_{02,21}(z')$. The indices 0, 1, 2 refer to different parton lines.

- To keep quark contributions, must also take N_f large.
- ➡ Must distinguish between dipoles made of actual quarks vs. large N_c gluons.
- ➡ Evolution equation closes, but even more complicated....

Outlook: The Truth Is Out There!

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- What about **other polarization observables** like **transversity**?

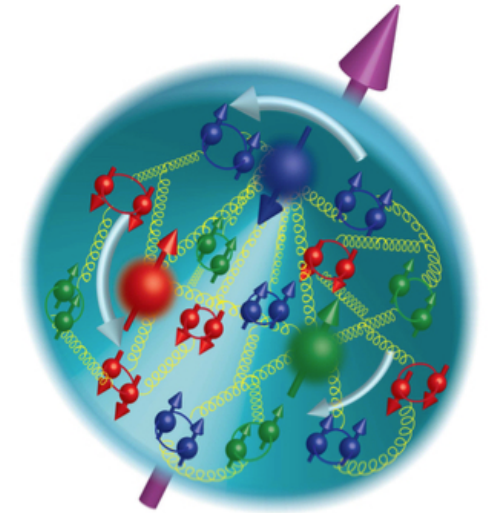
Summary

- Up to 35% of the proton angular momentum is unaccounted for.
➔ Is there significant polarization at small x ?

$$0.001 < x < 1$$

$$\Delta\Sigma \approx 0.25 \text{ (25\%)}$$

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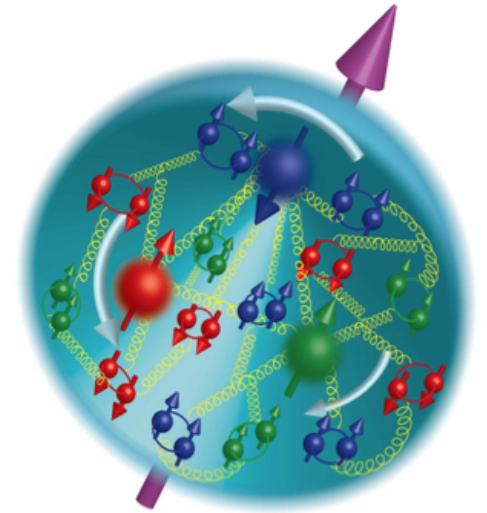
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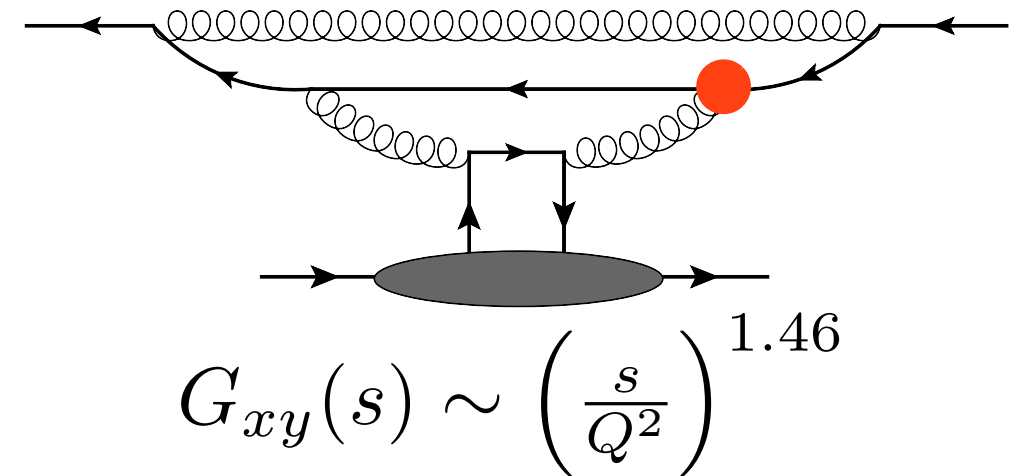
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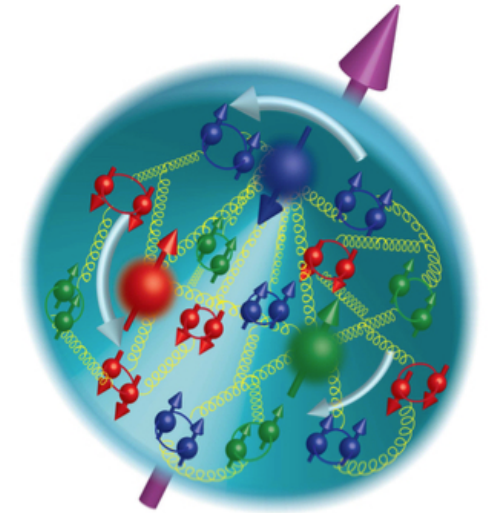
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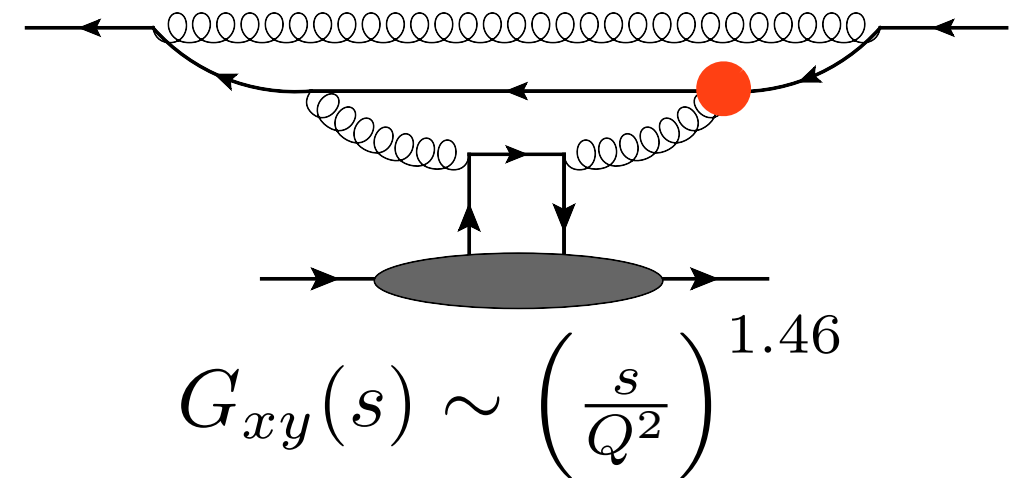
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- Massive complications due to non-ladder gluons and IR phase space.
➡ Much more to discover just around the corner!

